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Dynamical Symmetry Breaking And Electroweak Processes

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DYNAMICAL SYMMETRY BREAKING AND ELECTROWEAK PROCESSES

by

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ABSTRACT

The on-mass-shell renormalization prescription of electroweak theory is extended to account for shifts in the mass-shell. These shifts arise from dynamical contributions generated by the nonperturbative content of the vacuum to the quark self-energies. Upper limits for the value of the dimension-3 fermion-antifermion condensate are found by considering its contribution to the u-d mass difference and the strangeness-changing nonleptonic decays of the kaon. Dynamical generation of fermion masses under chiral gauge interactions is also examined by considering the coupling of light fermions to a heavy fermion-antifermion condensate.

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CHAPTER ONE
INTRODUCTION

There exist two different mass scales for $SU(3)_f$ quarks characterizing hadronic phenomenology. Quark masses that characterize current algebra are of the of order 0.005–0.010 GeV for nonstrange quarks, and 0.130–0.200 GeV for strange quarks.[1] On the other hand, static hadron properties, such as masses and magnetic moments, are successfully modelled[2] using nonstrange quark masses of 0.340 GeV, which is approximately 1/3 of a nucleon mass, and strange quark masses of 0.510 GeV, which is about 1/2 of the mass of the $s\bar{s}$ particle $\phi(1020)$. The near–leptonic scale of the former set of masses and methodological considerations from current algebra suggest that the former set of current masses correspond to masses in a field–theoretical Lagrangian. The latter set of constituent masses, which are scaled to the masses of known hadrons, are obtained from the former set by dynamical symmetry breaking. This dynamical symmetry breaking is the result of the failure of the QCD vacuum to respect the $SU(2)_f$ global chiral symmetry of the Lagrangian.

In considering electroweak interaction effects, one must take into account the chiral noninvariance of the $SU(3)_C \times SU(2)_L \times U(1)$ vacuum state, which leads to the formation of vacuum condensates that characterize low–energy QCD phenomenology. Such condensates necessarily couple to the electroweak interactions as well because quarks are fundamental fields of both QCD and standard–model $SU(2)_L \times U(1)$ electroweak physics. Therefore, it is necessary to include quark condensates, which arise from QCD, in quark self-energies mediated by the weak interaction gauge bosons.

Renormalization of electroweak theory, in isolation, involves the on-mass-shell renormalization of the fundamental fermion fields of the theory, leptons *and* quarks.[3] It is important to note that electroweak theory, in isolation, does not distinguish between quarks and leptons, permitting the consideration of mass-shell subtractions for both sets of fermions. Distinctions between quarks and leptons arise only from QCD, particularly its nonperturbative content, and lead both to confinement and to the constituent masses of the quarks that characterize static hadron properties.

Confinement is a long-distance property that arises from the nonperturbative regime of QCD. Despite infrared complexities, purely perturbative QCD will support a quark propagator pole to at least two-loop order.[4] However, the nonperturbative content of the QCD vacuum is also *perturbatively* coupled to perturbative $SU(2) \times U(1) \times SU(3)_c$ known interaction physics. Indeed the perturbative coupling of QCD-vacuum condensates to current-current correlation functions is the basis of QCD sum rule methodology.[5] Studies of QCD-vacuum condensate contributions to the quark two-point function fail to show any evidence for confinement, but have been argued to be responsible for shifts in the location of the quark propagator pole suggestive of a constituent mass.[6,7,8,9]

Since the true $SU(2) \times U(1) \times SU(3)_c$ vacuum state has nonperturbative content, any meaningful on-mass-shell renormalization prescription involving quark fields must take into account dynamical contributions to the self-energy function, which necessarily alter the location of the quark mass shell that is appropriate for the subtractions in electroweak renormalization.[8]

This thesis is concerned with extracting as much information about the quark mass as possible. Consequently, the renormalization of the $SU(3)_c \times SU(2) \times U(1)$ standard model is addressed in the presence of dynamical contributions to the quark mass matrix. Specifically, this thesis addresses whether an on-shell renormalization prescription can be extended to account for the presence of dynamical self-energy contributions; that is, can on-shell renormalization occur on a mass shell *differing* from the renormalized mass appearing in the Lagrangian?

In Chapter 2 the structure of the inverse fermion propagator matrix is discussed. The on-mass-shell renormalization conditions that are appropriate for a mass shifted from the perturbatively-renormalized Lagrangian mass are presented.

In Chapter 3 these conditions are perturbatively linearized, as would be appropriate for a self-energy contribution generated by the perturbative coupling to nonperturbative vacuum effects, i.e., QCD-vacuum condensates. It is also shown that the number of renormalization conditions obtained is equal to the number of renormalization constants available. This is a necessary condition for renormalizability.

In Chapter 4, the mass renormalization equation is discussed. The ambiguity in the definition of the mass counterterm is exploited to obtain a useful equation relating the difference between the current and constituent mass to the appropriate self-energies.

In Chapter 5, the results of the previous Chapters are applied to the quark-antiquark condensate arising from the nonperturbative sector of QCD. Specifically, the contribution of the dimension-3 condensate to the quark two-point function is considered.[9,10] The result obtained is shown, upon renormalization group improvement, to be consistent with the dynamical mass function obtained from integral equations for QCD Green's functions.[7]

In Chapter 6, the nonperturbative fermion propagator,[11] which is an expression valid *to all orders* in the fermion mass for the quark-condensate component of the normal-ordered ground state expectation value of fermion and antifermion fields, is employed to determine the dimension-3 quark condensate contributions to the quark two-point function that are mediated by electroweak particles. As one would expect, these additional electroweak contributions to the dynamical quark mass are negligible compared to the QCD contribution mediated by a gluon. This is a consequence of the small size of the known fermion masses and the QCD-vacuum condensates relative to the masses of the $W^{\pm}$ and Z.

In Chapters 7, 8, and 9, the on-mass-shell methodology of the previous Chapters is extended to some ideas that go beyond the standard model. Recent proposals of dynamical electroweak symmetry breaking driven by a $\langle\bar{t}t\rangle$ condensate[12,13] necessarily involve a condensate of enormous magnitude, at least of order μ_t^3. In

Chapter 7 the implications are examined of having a large $\langle \bar{t}t \rangle$ condensate that can be coupled, through off-diagonal weak currents, to d-and s-quark self-energies. Specifically, the expression for $\langle \tilde{0} | : t(x)\bar{t}(0) : | \tilde{0} \rangle$, which occurs in the Wick-Dyson expansion of the d-quark two-point function, is examined to estimate the $\langle \bar{t}t \rangle$ contribution to the d-quark mass. Terms such as $\langle \tilde{0} | : t(x)\bar{t}(0) : | \tilde{0} \rangle$ would be zero in a purely perturbative vacuum. Because $\langle \bar{t}t \rangle$ is *not* coupled to the u-quark two-point function by electroweak currents, the $\langle \bar{t}t \rangle$ contribution to the d-quark mass should not exceed the u-d mass difference. If the contribution did exceed this difference then fine-tuning of the Lagrangian masses would be needed to compensate for the resulting disparity. This restriction places an upper bound on the magnitude of $\langle \bar{t}t \rangle$, which, though quite large, is well below the magnitudes suggested by four-fermion interactions referenced to Planck momentum scales.[13]

In Chapter 8 the contribution of $\langle \bar{t}t \rangle$ to the on-mass-shell renormalized *sd* component of the inverse propagator matrix is examined. This off-diagonal contribution is known to be quite small within purely perturbative contexts because of simultaneous s-and d-mass shell subtractions.[14] However, it is sensitive to the size of the $\langle \bar{t}t \rangle$ condensate. A very large t-quark condensate may yield a sufficiently large renormalized *sd* inverse-propagator component to account for the observed $\Delta I = 1/2$ enhancement of strangeness-changing nonleptonic weak decays. It should be emphasized that the results of Chapters 7 and 8 are applicable to *any* dynamical symmetry breaking scenario in which a very large fermion-antifermion condensate is accessible to quarks through an interaction that involves an off–diagonal gauge interaction.

Finally, in Chapter 9 the overall issue of dynamical mass generation is considered for the general case of a fermion that is protected by Lagrangian symmetries from acquiring a mass perturbatively. If such a fermion experiences only chiral gauge interactions, then it can be demonstrated that the on–mass–shell renormalization conditions of Chapter 2 provide for both a purely perturbative zero–mass solution, and a nonperturbative nonzero–mass solution. The compatibility of the latter solution, corresponding to the dynamical generation of a fermion mass, is discussed with respect to a scenario in which off–diagonal interactions couple the fermion in question to a large fermion–antifermion condensate of a different flavour. The nonzero–mass solution leads to an induced fermion mass *inversely proportional* to the condensate. Hence, gauge symmetry–breaking effected by very large condensates may also serve to induce appropriate mass scales for at least some of the known fundamental fermion fields.

The ideas presented in these last three sections are, of course, quite speculative. The presentation of them is intended neither to promote nor dispute any particular model for dynamical symmetry breaking, but rather to illustrate the utility of techniques that up until now have been applied only within the context of QCD sum–rule physics.[11,15,16,17] Specifically, much use has been made of the nonperturbative propagator, which is valid to all orders for the $\langle \overline{f} f \rangle$ projection of the $\langle \tilde{0} | : f(x)\overline{f}(0) : | \tilde{0} \rangle$ ground state expectation value [equations (7.2.8), (C.1), and (C.2)] developed independently in References 9, 11, and 15. Incorporation of this expression into vector-current vacuum polarization functions[16] and the VVA-triangle diagram[18] upholds the requisite vector and axial-vector Ward identities at all momentum scales. Incorporation of the nonperturbative propagator into the axial-vector-current correlation function has also been explored. A QCD sum-rule analysis involving a contour integration over the region of physical momentum is consistent with sum-rule results obtained by standard methods in the deep Euclidean region.[19] Applicability of the nonperturbative propagator to symmetry-breaking scenarios that are driven by fermion-antifermion condensates merits exploration, even if only to stimulate debate and fresh ideas on the origin of symmetry breaking in the fundamental forces of nature.

CHAPTER TWO
ELECTROWEAK THEORY AND THE MASS MATRIX

2.1 Electroweak Theory and the Higgs Mechanism

The electroweak theory is described by an $SU(2) \times U(1)$ gauge theory.[20] The $SU(2)$ algebra is generated by the three generators T^1, T^2 and T^3. The $U(1)$ algebra is generated by the single generator Y, which is also called the hypercharge for historical reasons and is related to the charge operator by the relation $Q = T^3 + Y/2$. The generators satisfy the commutation relations

$$[T^a, T^b] = i\epsilon^{abc}T^c\,, \tag{2.1.1}$$

$$[T^a, Y] = 0\,. \tag{2.1.2}$$

The quarks are collected into left–handed SU(2) doublets and right–handed U(1) singlets,

$$L'_I = \begin{pmatrix} \psi'_I \\ \widetilde{\psi}'_I \end{pmatrix}_L\,, \quad \widetilde{R}'_I = (\widetilde{\psi}'_I)_R\,, \quad R'_I = (\psi'_I)_R\,. \tag{2.1.3}$$

The prime indicates that the field is a gauge eigenstate. The fields without a tilde represent the charge +2/3 quarks, u, c and t. The fields with the tilde represent the corresponding charge −1/3 quarks, d, s and b in their respective doublets. For instance, ψ'_u is the gauge eigenstate for the up–quark and $\widetilde{\psi}'_u$ is the gauge eigenstate for the down–quark.

In order to define a gauge invariant Lagrangian, one must define covariant derivatives that transform the same way as the fields themselves. The covariant

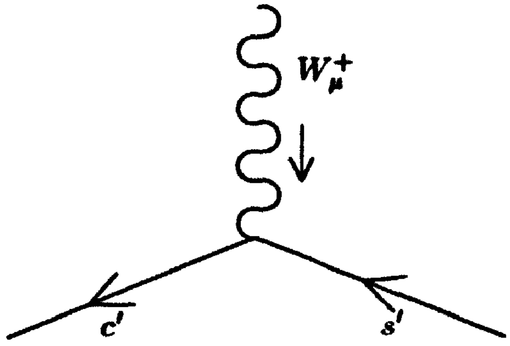


Figure 2.1 In terms of gauge eigenstates, an s'–quark can transform only into a c'–quark by interacting with a W_μ^+ gauge boson.

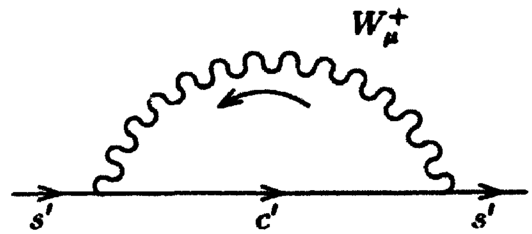


Figure 2.2 In terms of gauge eigenstates, an s'–quark can transform only into an s'–quark by a self–energy interaction involving a W_μ^+.

derivative that operates on the left–handed doublets is

$$D_{L\mu} = \partial_\mu - ig\sum_{a=1}^{3} T^a W_\mu^a - \frac{1}{2} ig' Y W_\mu^0 , \qquad (2.1.4)$$

while the covariant derivative that operates on the right–handed singlets is

$$D_{R\mu} = \partial_\mu - \frac{1}{2} ig' Y W_\mu^0 . \qquad (2.1.5)$$

The part of the Lagrangian that contains the quarks and is invariant under an $SU(2) \times U(1)$ gauge transformation is

$$\mathcal{L}_F = i\sum_I \overline{L}'_I \not{D}_L L'_I + i\sum_I \overline{R}'_I \not{D}_R R'_I + i\sum_I \overline{\tilde{R}}'_I \not{D}_R \tilde{R}'_I . \qquad (2.1.6)$$

This Lagrangian is invariant under a gauge transformation of the primed fields, which is why they are called the gauge eigenstates. It can be separated into a kinetic term and an interaction term:

$$\begin{aligned}\mathcal{L}_F = i\sum_I \overline{L}'_I \not{\partial} L_I + i\sum_I \overline{R}'_I \not{\partial} R_I + i\sum_I \overline{\tilde{R}}'_I \not{\partial} \tilde{R}_I & \\ + \frac{1}{2}\sum_I \overline{L}'_I \gamma^\mu \begin{pmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{pmatrix} L'_I + \ldots . & \qquad (2.1.7)\end{aligned}$$

Note that the interaction allows an up–type quark to transform into a down–type quark by interacting with the charged gauge bosons $W_\mu^\pm = W_\mu^1 \mp iW_\mu^2$. However, quarks from different generations do not mix. That is, a charm quark can transform into a strange quark and an up quark can transform into a down quark, but a charm quark cannot transform into a down quark. So, for instance, an s′–quark can interact with a W^+ and become a c′–quark (Figure 2.1). If the c′–quark subsequently absorbs the W^+, then the only thing it can become is a s′–quark (Figure 2.2). Therefore, because the interaction Lagrangian, written in terms of the gauge eigenstates, does not allow intergenerational mixing of the quarks, it is not possible to have off–diagonal self–energies, which means there are no off–diagonal elements in the propagator. It will be shown later that the diagonalization of the mass matrix causes intergenerational mixing when the Lagrangian is written in terms of the mass eigenstates.

The quarks acquire mass by coupling to the Higgs field Φ in the following way:

$$\mathcal{L}^{\text{Higgs}} = -\sum_{I,J} f_{IJ}\overline{L}'_I(i\tau_2\Phi^*)R'_J - \sum_{I,J} \widetilde{f}_{IJ}\overline{\widetilde{L}}'_I\Phi\widetilde{R}'_J + \text{h.c.}, \tag{2.1.8}$$

where f_{IJ} and $\widetilde{f}_{IJ}$ are the Yukawa coupling constants for the up–type and down–type quarks, respectively. As will be shown below, the fact that the Yukawa coupling is not diagonal leads to a redefinition of the quark fields in terms of mass eigenstates. The Higgs field is a complex doublet:

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} i\chi_1(x) + \chi_2(x) \\ v + \phi(x) - i\chi_3(x) \end{pmatrix}. \tag{2.1.9}$$

The real parameter v is the vacuum expectation value of the Higgs field

$$\langle 0|\Phi|0\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} . \tag{2.1.10}$$

This nonzero vacuum expectation value allows the quarks to gain mass. To see this, substitute (2.1.9) into (2.1.8) to obtain

$$\mathcal{L}^{\text{Higgs}} = -\sum_{I,J} \frac{v}{\sqrt{2}} f_{IJ} \overline{\psi}'_{LI} \psi'_{RJ} - \sum_{I,J} \frac{v}{\sqrt{2}} \tilde{f}_{IJ} \overline{\tilde{\psi}}'_{LI} \tilde{\psi}'_{RJ} + \text{h.c.} + \mathcal{L}^{\text{Higgs}}_{\text{int}} , \tag{2.1.11}$$

where $\mathcal{L}^{\text{Higgs}}_{\text{int}}$ contains the interaction terms between the components of the Higgs field χ_i and ϕ and the quark fields. The mass matrices are defined by

$$M_{IJ} = \frac{v}{\sqrt{2}} f_{IJ} , \quad \widetilde{M}_{IJ} = \frac{v}{\sqrt{2}} \tilde{f}_{IJ} . \tag{2.1.12}$$

There are off–diagonal terms in the mass matrix because the Yukawa coupling is off–diagonal. The eigenvalues of the mass matrix are the masses of the quarks, and its eigenvectors are the mass eigenstates. The Lagrangian can be rewritten in terms of the mass eigenstates.

2.2 Diagonalization of the Mass Matrix

In this section, the eigenvectors of the mass matrix are found and the Lagrangian is rewritten in terms of these mass eigenstates.[21]

Spontaneous symmetry breaking in electroweak theory produces mass matrices that are not diagonal. These mass matrices connect quarks that have the same charge. The terms in (2.1.11) that contain the masses are

$$\mathcal{L}_{\mathrm{M}}^{\mathrm{Higgs}} = -\left(\overline{\psi'}_u \quad \overline{\psi'}_c \quad \overline{\psi'}_t\right)_L \begin{pmatrix} m_{uu} & m_{uc} & m_{ut} \\ m_{cu} & m_{cc} & m_{ct} \\ m_{tu} & m_{tc} & m_{tt} \end{pmatrix} \begin{pmatrix} \psi'_u \\ \psi'_c \\ \psi'_t \end{pmatrix}_R$$

$$-\left(\overline{\widetilde{\psi}'}_u \quad \overline{\widetilde{\psi}'}_c \quad \overline{\widetilde{\psi}'}_t\right)_L \begin{pmatrix} \widetilde{m}_{uu} & \widetilde{m}_{uc} & \widetilde{m}_{ut} \\ \widetilde{m}_{cu} & \widetilde{m}_{cc} & \widetilde{m}_{ct} \\ \widetilde{m}_{tu} & \widetilde{m}_{tc} & \widetilde{m}_{tt} \end{pmatrix} \begin{pmatrix} \widetilde{\psi}'_u \\ \widetilde{\psi}'_c \\ \widetilde{\psi}'_t \end{pmatrix}_R + \text{h.c.}\,. \qquad (2.2.1)$$

The mass matrices M and $\widetilde{M}$ are, in general, neither symmetric nor hermitian.

In order to find the mass eigenstates, the mass matrices must be diagonalized. This is accomplished by a biunitary transformation

$$S^{\dagger}MT = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}, \quad \widetilde{S}^{\dagger}\widetilde{M}\widetilde{T} = \begin{pmatrix} \widetilde{m}_d & 0 & 0 \\ 0 & \widetilde{m}_s & 0 \\ 0 & 0 & \widetilde{m}_b \end{pmatrix}, \qquad (2.2.2)$$

where S, T, $\widetilde{S}$ and $\widetilde{T}$ are 3×3 unitary matrices. These matrices can be inserted into equation (2.2.1):

$$\mathcal{L}_{\mathrm{M}}^{\mathrm{Higgs}} = -\left(\overline{\psi'}_u \quad \overline{\psi'}_c \quad \overline{\psi'}_t\right)_L SS^{\dagger} \begin{pmatrix} m_{uu} & m_{uc} & m_{ut} \\ m_{cu} & m_{cc} & m_{ct} \\ m_{tu} & m_{tc} & m_{tt} \end{pmatrix} TT^{\dagger} \begin{pmatrix} \psi'_u \\ \psi'_c \\ \psi'_t \end{pmatrix}_R$$

$$-\left(\overline{\widetilde{\psi}'}_u \quad \overline{\widetilde{\psi}'}_c \quad \overline{\widetilde{\psi}'}_t\right)_L \widetilde{S}\widetilde{S}^{\dagger} \begin{pmatrix} \widetilde{m}_{uu} & \widetilde{m}_{uc} & \widetilde{m}_{ut} \\ \widetilde{m}_{cu} & \widetilde{m}_{cc} & \widetilde{m}_{ct} \\ \widetilde{m}_{tu} & \widetilde{m}_{tc} & \widetilde{m}_{tt} \end{pmatrix} \widetilde{T}\widetilde{T}^{\dagger} \begin{pmatrix} \widetilde{\psi}'_u \\ \widetilde{\psi}'_c \\ \widetilde{\psi}'_t \end{pmatrix}_R + \text{h.c.}\,. \qquad (2.2.3)$$

In this way the mass matrices are diagonalized. But now the left-handed and right-handed fields must be redefined in order to absorb the unitary matrices S, T, $\widetilde{S}$ and $\widetilde{T}$:

$$\begin{pmatrix} \psi'_u \\ \psi'_c \\ \psi'_t \end{pmatrix}_L = S \begin{pmatrix} \psi_u \\ \psi_c \\ \psi_t \end{pmatrix}_L, \quad \begin{pmatrix} \psi'_u \\ \psi'_c \\ \psi'_t \end{pmatrix}_R = T \begin{pmatrix} \psi_u \\ \psi_c \\ \psi_t \end{pmatrix}_R, \qquad (2.2.4)$$

$$\begin{pmatrix} \widetilde{\psi}'_u \\ \widetilde{\psi}'_c \\ \widetilde{\psi}'_t \end{pmatrix}_L = \widetilde{S} \begin{pmatrix} \widetilde{\psi}_u \\ \widetilde{\psi}_c \\ \widetilde{\psi}_t \end{pmatrix}_L, \quad \begin{pmatrix} \widetilde{\psi}'_u \\ \widetilde{\psi}'_c \\ \widetilde{\psi}'_t \end{pmatrix}_R = \widetilde{T} \begin{pmatrix} \widetilde{\psi}_u \\ \widetilde{\psi}_c \\ \widetilde{\psi}_t \end{pmatrix}_R. \qquad (2.2.5)$$

The primed fields are the gauge eigenstates and the unprimed fields are the mass eigenstates. The kinetic and neutral current terms are invariant under this redefinition. On the other hand, this redefinition has important consequences for the interaction of the quarks with the charged gauge bosons $W^{\pm}$. Consider the charged current coupled to the W_{μ}^{+} from (2.1.7) in terms of the gauge eigenstates

$$J^{\mu}W_{\mu}^{+} = \frac{1}{2}\sum_{I}\overline{\psi}'_{LI}\gamma^{\mu}\widetilde{\psi}'_{LI}W_{\mu}^{+}\,. \tag{2.2.6}$$

Written in terms of the mass eigenstates this becomes

$$J^{\mu}W_{\mu}^{+} = \frac{1}{2}\sum_{IJ}\overline{\psi}_{LI}\gamma^{\mu}[S^{\dagger}\widetilde{S}]_{IJ}\widetilde{\psi}_{LJ}W_{\mu}^{+}\,. \tag{2.2.7}$$

Now another unitary operator is defined:

$$V = S^{\dagger}\widetilde{S}\,. \tag{2.2.8}$$

This is the Kobayashi–Maskawa matrix.[22] It will prove useful in the rest of this work to use a different notation. Instead of denoting the down quark, say, by $\widetilde{\psi}_{u}$ it will be denoted by ψ_{d}. In general the lower components of the $SU(2)$ doublet will be denoted by a lower–case letter. Thus, the charged current (2.2.7) can be rewritten as

$$J^{\mu}W_{\mu}^{+} = \frac{1}{2}\sum_{I=u,c,t}\sum_{i=d,s,b}\overline{\psi}_{LI}\gamma_{\mu}V_{Ii}\psi_{Li}W_{\mu}^{+}\,. \tag{2.2.9}$$

With this change in notation, the doublets of $SU(2)$ are changed. The upper component of the doublet is the mass eigenstate but the lower component is now a mixture of the mass eigenstates:

$$\begin{pmatrix}\psi_{u}\\ \psi_{u}\end{pmatrix} = \begin{pmatrix}\psi_{u}\\ V_{ud}\psi_{d}+V_{us}\psi_{s}+V_{ub}\psi_{b}\end{pmatrix}\,. \tag{2.2.10}$$

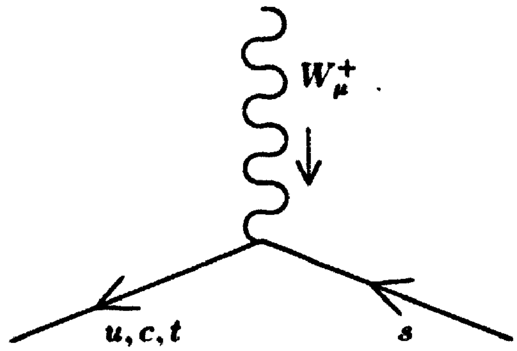


Figure 2.3 In terms of mass eigenstates, an s–quark can transform into a u–,c–or t–quark by interacting with a W_μ^+ gauge boson.

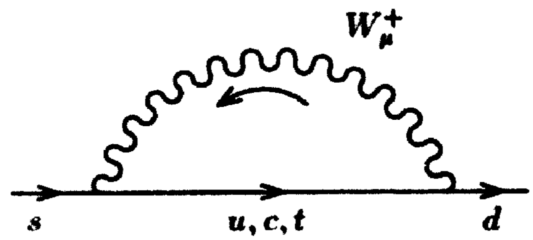


Figure 2.4 In terms of mass eigenstates, an s–quark can transform into a d–quark by a self–energy interaction involving a W_μ^+.

In general,

$$\begin{pmatrix} \tilde{\psi}_I \\ \psi_I \end{pmatrix} = \begin{pmatrix} \psi_I \\ \sum_{i=d,s,b} V_{Ii}\psi_i \end{pmatrix} . \tag{2.2.11}$$

Now it is possible for a c–quark to transform into either a d–, s–or b–quark by interacting with a $W^\pm$. So, for instance, an s–quark can shake off a $W^\pm$ and become either a u–, c–or t–quark (Figure 2.3). If this quark subsequently absorbs the $W^\pm$, it can become either a d–, s–or b–quark. If it becomes, say, a d–quark (Figure 2.4), then the whole process is an off–diagonal self–energy transition. This means that it is possible to have off–diagonal elements in the propagator.

When the Lagrangian is written in terms of the gauge eigenstates, the mass matrix is off–diagonal but the interaction term allows only diagonal propagation. Now that the Lagrangian is written in terms of the mass eigenstates, the mass matrix is diagonal but the interaction term allows off–diagonal propagation.

2.3 Diagonalization of the Inverse Propagator

Because of the presence of off-diagonal self-energies, there are multiple poles in the propagator. This can be seen by considering the perturbative expansion for the full propagator S'_{ij}

$$S'_{ij} = \delta_{ij} S_i + \sum_{k=1}^{N} S'_{ik} \Pi_{kj} S_j \qquad i,j = 1,2,\ldots,N\,, \tag{2.3.1}$$

where i and j are flavour indices, $S_i = 1/(m_i - \not{p})$ is the free propagator for the i^{th} particle and Π_{ij} is the one-particle-irreducible self-energy. By repeated iteration of (2.3.1), one obtains

$$S'_{ij} = \delta_{ij} S_i + S_i \Pi_{ij} S_j + \sum_{k=1}^{N} S_i \Pi_{ik} S_k \Pi_{kj} S_j + \ldots\,. \tag{2.3.2}$$

A diagonal propagator $i = j$ has poles at each mass m_k because of the third term. Thus, the propagator cannot be identified with the propagation of any particular particle. Such an identification requires that the propagator has a unique pole, which is identified with the particle's mass. To interpret the propagators as the propagators of physical particles, one must diagonalize S'_{ij}.[23,24] In fact, in Chapter 3 it is the inverse propagator that is diagonalized. If the inverse propagator is diagonal, then the propagator will also be diagonal. The matrices that diagonalize the inverse propagator are just the wavefunction renormalization constants. Thus, diagonalization is accomplished through renormalization.

CHAPTER THREE
RENORMALIZATION AND LINEARIZATION

3.1 On–Mass–Shell Renormalization of the Electroweak Theory

In computing the propagator in any quantum field theory, one encounters divergences that must be eliminated by renormalization. In this section the renormalization conditions are formulated in terms of the inverse propagator following the approach of Reference 3. The conditions are imposed at a point μ_i that differs from the Lagrangian · ıass m_i.

Consider the kinetic part of the Lagrangian for the quarks,

$$\mathcal{L} = \overline{\Psi}^0 (i\partial\!\!\!/ - \mathbf{M}^0)\Psi^0 + \mathcal{L}_{\text{int}} \,. \tag{3.1.1}$$

where special attention will be given to terms bilinear in fermion fields. For simplicity only the charge $-1/3$ quarks will be considered. The procedure for the renormalization of charge $+2/3$ quark fields is identical. The object Ψ^0 is a column matrix whose elements are fermionic fields that are all of the same charge and colour,

$$\Psi^0 = \begin{pmatrix} \psi_1^0 \\ \psi_2^0 \\ \vdots \\ \psi_N^0 \end{pmatrix} , \tag{3.1.2}$$

where N is the number of families. The mass matrix $\mathbf{M}^0$ is an $N \times N$, diagonal matrix

$$\mathbf{M}^0 = \begin{pmatrix} m_1^0 & 0 & \dots & 0 \\ 0 & m_2^0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & m_N^0 \end{pmatrix} . \tag{3.1.3}$$

The bare mass is the difference between the renormalized Lagrangian mass m_i and the mass counterterm, $m_i^0 = m_i - \delta m_i$.

Electroweak theory does not respect parity. Therefore, the left–and right–handed components of the fields must be treated independently. The field can be split into its left–and right–handed components with the use of the projection operators $L = (1 - \gamma_5)/2$ and $R = (1 + \gamma_5)/2$:

$$\Psi^0 = L\Psi^0 + R\Psi^0 . \tag{3.1.4}$$

The left– and right–components must be renormalized separately. Because of the intergenerational mixing caused by the diagonalization of the mass matrix, the propagator will have off–diagonal elements as discussed in Chapter 2. Such off–diagonal transitions must vanish as one approaches the mass–shell. Therefore, the propagator must be diagonalized. If the bare fields are written as combinations of the renormalized fields

$$L\Psi^0 = \mathbf{Z}_L^{1/2} L\Psi , \qquad R\Psi^0 = \mathbf{Z}_R^{1/2} R\Psi , \tag{3.1.5}$$

where $\mathbf{Z}_{R,L}^{1/2}$ are $N \times N$ matrices with complex elements, then the imposition of appropriate renormalization conditions will result in the diagonalization of the propagator:

$$\Psi^0 = (\mathbf{Z}_L^{1/2} L + \mathbf{Z}_R^{1/2} R)\Psi , \qquad \overline{\Psi}^0 = \overline{\Psi}(\mathbf{Z}_L^{1/2\dagger} R + \mathbf{Z}_R^{1/2\dagger} L) . \tag{3.1.6}$$

Substitute (3.1.6) back into (3.1.1) to obtain

$$\mathcal{L} = \overline{\Psi}\Big[-\frac{1}{2}\Big(\mathbf{Z}_L^{1/2\,\dagger}\mathbf{M}_0\mathbf{Z}_R^{1/2} + \mathbf{Z}_R^{1/2\,\dagger}\mathbf{M}_0\mathbf{Z}_L^{1/2}\Big)\mathbf{1} + \frac{1}{2}\Big(\big|\mathbf{Z}_L^{1/2}\big|^2 + \big|\mathbf{Z}_R^{1/2}\big|^2\Big)i\partial\!\!\!/ -$$
$$-\frac{1}{2}\Big(\mathbf{Z}_L^{1/2\,\dagger}\mathbf{M}_0\mathbf{Z}_R^{1/2} - \mathbf{Z}_R^{1/2\,\dagger}\mathbf{M}_0\mathbf{Z}_L^{1/2}\Big)\gamma_5 + \frac{1}{2}\Big(\big|\mathbf{Z}_L^{1/2}\big|^2 - \big|\mathbf{Z}_R^{1/2}\big|^2\Big)\gamma_5 i\partial\!\!\!/\Big]\Psi +$$
$$+\mathcal{L}_{\text{int}}\,. \tag{3.1.7}$$

We can immediately obtain the inverse propagator in momentum space:

$$\mathbf{K}(p\!\!\!/) = -\frac{1}{2}\Big(\mathbf{Z}_L^{1/2\,\dagger}\mathbf{M}_0\mathbf{Z}_R^{1/2} + \mathbf{Z}_R^{1/2\,\dagger}\mathbf{M}_0\mathbf{Z}_L^{1/2}\Big)\mathbf{1} + \frac{1}{2}\Big(\big|\mathbf{Z}_L^{1/2}\big|^2 + \big|\mathbf{Z}_R^{1/2}\big|^2\Big)p\!\!\!/ -$$
$$-\frac{1}{2}\Big(\mathbf{Z}_L^{1/2\,\dagger}\mathbf{M}_0\mathbf{Z}_R^{1/2} - \mathbf{Z}_R^{1/2\,\dagger}\mathbf{M}_0\mathbf{Z}_L^{1/2}\Big)\gamma_5 + \frac{1}{2}\Big(\big|\mathbf{Z}_L^{1/2}\big|^2 - \big|\mathbf{Z}_R^{1/2}\big|^2\Big)\gamma_5 p\!\!\!/ +$$
$$+\boldsymbol{\Sigma}_1(p^2)\mathbf{1} + \boldsymbol{\Sigma}_\gamma(p^2)p\!\!\!/ + \boldsymbol{\Sigma}_5(p^2)\gamma_5 + \boldsymbol{\Sigma}_{5\gamma}(p^2)\gamma_5 p\!\!\!/\,. \tag{3.1.8}$$

The free inverse propagator in coordinate space has the form

$$K^0(x) = i\partial\!\!\!/ - m\,. \tag{3.1.9}$$

A number of conditions exist that relate the components of the inverse propagator. These conditions substantially reduce the number of independent renormalization equations. These conditions are derived from the Hermiticity of the dressed inverse propagator. It is easily shown that the operator $\gamma_0 K^0(x)$ is Hermitian, i.e., it satisfies the condition

$$\int \mathrm{d}^4x\,\big[\gamma_0 K^0(x)\psi(x)\big]^\dagger\psi(x) = \int \mathrm{d}^4x\,\psi(x)^\dagger\big[\gamma_0 K^0(x)\psi(x)\big]\,. \tag{3.1.10}$$

Transforming to momentum space, one obtains the condition

$$K^0(p\!\!\!/) = \gamma_0 K^0(p\!\!\!/)^\dagger\gamma_0\,. \tag{3.1.11}$$

It is assumed that the full dressed propagator (3.1.8) must satisfy an identical condition:

$$\mathbf{K}(\not p) = \gamma_0 \mathbf{K}(\not p)^\dagger \gamma_0 \,. \tag{3.1.12}$$

Given this condition, the self-energies must satisfy the following relations:

$$\mathbf{\Sigma_1}(p^2)^\dagger = \quad \mathbf{\Sigma_1}(p^2)\,, \tag{3.1.13}$$

$$\mathbf{\Sigma_\gamma}(p^2)^\dagger = \quad \mathbf{\Sigma_\gamma}(p^2)\,, \tag{3.1.14}$$

$$\mathbf{\Sigma_5}(p^2)^\dagger = -\mathbf{\Sigma_5}(p^2)\,, \tag{3.1.15}$$

$$\mathbf{\Sigma_{5\gamma}}(p^2)^\dagger = \quad \mathbf{\Sigma_{5\gamma}}(p^2)\,. \tag{3.1.16}$$

The inverse propagator can be written as

$$\mathbf{K}(\not p) = \mathbf{K_1}(p^2)\mathbf{1} + \mathbf{K_\gamma}(p^2)\not p + \mathbf{K_5}(p^2)\gamma_5 + \mathbf{K_{5\gamma}}(p^2)\gamma_5\not p\,, \tag{3.1.18}$$

where the components are defined to be[3]

$$\mathbf{K_1}(p^2) = -\frac{1}{2}\left(\mathbf{Z}_L^{1/2\,\dagger}\mathbf{M}_0\mathbf{Z}_R^{1/2} + \mathbf{Z}_R^{1/2\,\dagger}\mathbf{M}_0\mathbf{Z}_L^{1/2}\right) + \mathbf{\Sigma_1}(p^2)\,, \tag{3.1.19}$$

$$\mathbf{K_\gamma}(p^2) = \quad \frac{1}{2}\left(\left|\mathbf{Z}_L^{1/2}\right|^2 + \left|\mathbf{Z}_R^{1/2}\right|^2\right) + \mathbf{\Sigma_\gamma}(p^2)\,, \tag{3.1.20}$$

$$\mathbf{K_5}(p^2) = -\frac{1}{2}\left(\mathbf{Z}_L^{1/2\,\dagger}\mathbf{M}_0\mathbf{Z}_R^{1/2} - \mathbf{Z}_R^{1/2\,\dagger}\mathbf{M}_0\mathbf{Z}_L^{1/2}\right) + \mathbf{\Sigma_5}(p^2)\,, \tag{3.1.21}$$

$$\mathbf{K_{5\gamma}}(p^2) = \quad \frac{1}{2}\left(\left|\mathbf{Z}_L^{1/2}\right|^2 - \left|\mathbf{Z}_R^{1/2}\right|^2\right) + \mathbf{\Sigma_{5\gamma}}(p^2)\,. \tag{3.1.22}$$

In order to maintain the CP–invariance of the theory, the on–diagonal terms of the propagator must be CP–invariant. All of the terms are CP–invariant except for the γ_5 term. Therefore, the on–diagonal γ_5 term must vanish:

$$K_{5,ii}(p^2) = 0 \qquad (\text{no sum over } i)\,. \tag{3.1.23}$$

Substitute (3.1.21) into (3.1.23) to obtain an equation that relates the renormalization constants,

$$\sum_{j=1}^{N} m_j^0 Z_{L,ij}^{1/2\,\dagger} Z_{R,ji}^{1/2} = \sum_{j=1}^{N} m_j^0 Z_{R,ij}^{1/2\,\dagger} Z_{L,ji}^{1/2} \qquad i = 1, 2, 3, \ldots , \tag{3.1.24}$$

as well as an equation eliminating the γ_5 term of the on–diagonal self–energy:

$$\Sigma_{5,ii}(p^2) = 0 . \tag{3.1.25}$$

These equations must be checked when the renormalization constants and the self–energies have been calculated.

Now the mass–shell renormalization conditions must be imposed on the inverse propagator.

3.2 The Renormalization Conditions

Following the methodology of Reference 3, the on–mass–shell renormalization prescription at μ_i is adopted. It is assumed that

$$\not{p}\psi_i(p) = \mu_i\psi_i(p) , \quad \overline{\psi}_i(p)\not{p} = \mu_i\overline{\psi}_i(p) . \tag{3.2.1}$$

The point μ_i is identified with the propagator pole, in which case the inverse propagator must vanish on–shell

$$K_{ii}(\not{p})\psi_i(p) = 0 . \tag{3.2.2}$$

The condition

$$\left[\frac{1}{\not{p} - \mu_i} K_{ii}(\not{p})\right]\psi_i = \psi_i \tag{3.2.3}$$

ensures that the propagator has unit residue at the pole μ_i.

For the off-diagonal terms, the following conditions are imposed:

$$K_{ij}(\not{p})\psi_j = 0\,, \tag{3.2.4}$$

$$\overline{\psi}_i K_{ij}(\not{p}) = 0\,. \tag{3.2.5}$$

These last two conditions guarantee that there is no off-diagonal propagation of free particles on the mass-shell. That is, whenever either of the particles is free, it cannot spontaneously propagate into another particle. But note that off-diagonal propagation may occur off-shell, i.e., in bound states.

Substitute (3.1.18) into (3.2.2) to obtain the first of the conditions on the diagonal elements of the inverse propagator:

$$K_{1,ii}(\mu_i^2) + \mu_i K_{\gamma,ii}(\mu_i^2) = 0\,. \tag{3.2.6}$$

$$K_{5\gamma,ii}(\mu_i^2) = 0\,. \tag{3.2.7}$$

Substitute (3.1.18) into (3.2.3) to obtain the additional condition:

$$\left[\frac{\not{p}+\mu_i}{p^2-\mu_i^2}\left[K_{1,ii}(p^2)1 + K_{\gamma,ii}(p^2)\not{p} + K_{5\gamma,ii}(p^2)\gamma_5\not{p}\right]\right]\psi_i = \psi_i\,. \tag{3.2.8}$$

In the second term substitute $\not{p} = (\not{p}-\mu_i)+\mu_i$ to obtain

$$\left[\frac{\not{p}+\mu_i}{p^2-\mu_i^2}\left\{\left[K_{1,ii}(p^2)+\mu_i K_{\gamma,ii}(p^2)\right]1 + K_{\gamma,ii}(p^2)(\not{p}-\mu_i) + K_{5\gamma,ii}(p^2)\gamma_5\not{p}\right\}\right]\psi_i = \psi_i\,. \tag{3.2.9}$$

Now expand the first and the last terms in the curly brackets into Taylor series about the point $p^2 = \mu_i$, and use (3.2.6) and (3.2.7) to obtain

$$\Big\{(\not{p} + \mu_i)\big[K'_{1,ii}(\mu_i^2) + \mu_i K'_{\gamma,ii}(\mu_i^2)\big] + (p^2 - \mu_i^2)(\not{p} + \mu_i)\big[\widetilde{K}_{1,ii}(p^2) + \mu_i \widetilde{K}_{\gamma,ii}(p^2)\big]$$

$$+ K_{\gamma,ii}(p^2)\mathbf{1} + (\not{p} + \mu_i)\big[K'_{5\gamma,ii}(\mu_i^2) + (p^2 - \mu_i^2)\widetilde{K}_{5\gamma,ii}(p^2)\big]\gamma_5 \not{p}\Big\}\psi_i = \psi_i \,. \quad (3.2.10)$$

The prime indicates the derivative with respect to p^2, and the tilde indicates the remainder of the Taylor series. From (3.2.1) one obtains the result

$$2\mu_i\big[K'_{1,ii}(\mu_i^2) + \mu_i K'_{\gamma,ii}(\mu_i^2)\big]\mathbf{1} + K_{\gamma,ii}(\mu_i^2)\mathbf{1} + 2\mu_i^2 K'_{5\gamma,ii}(\mu_i^2)\gamma_5 = \mathbf{1}\,. \quad (3.2.11)$$

The terms $\mathbf{1}$ and γ_5 are linearly independent, so (3.2.11) yields two equations:

$$K_{\gamma,ii}(\mu_i^2) + 2\mu_i\big[K'_{1,ii}(\mu_i^2) + \mu_i K'_{\gamma,ii}(\mu_i^2)\big] = 1\,, \quad (3.2.12)$$

$$K'_{5\gamma,ii}(\mu_i^2) = 0\,. \quad (3.2.13)$$

In a similar way (3.2.4) and (3.2.5) yield conditions on the off-diagonal components of the inverse propagator.

The on-diagonal conditions thus obtained are, altogether,

$$K_{1,ii}(\mu_i^2) + \mu_i K_{\gamma,ii}(\mu_i^2) = 0\,, \quad (3.2.14)$$

$$K_{5\gamma,ii}(\mu_i^2) = 0\,, \quad (3.2.15)$$

$$K_{\gamma,ii}(\mu_i^2) + 2\mu_i\big[K'_{1,ii}(\mu_i^2) + \mu_i K'_{\gamma,ii}(\mu_i^2)\big] = 1\,. \quad (3.2.16)$$

The off-diagonal conditions obtained from (3.2.4) and (3.2.5) are

$$K_{1,ij}(\mu_j^2) + \mu_j K_{\gamma,ij}(\mu_j^2) \;= 0\,, \qquad (3.2.17)$$

$$K_{5,ij}(\mu_j^2) + \mu_j K_{5\gamma,ij}(\mu_j^2) = 0\,, \qquad (3.2.18)$$

$$K_{1,ij}(\mu_i^2) + \mu_i K_{\gamma,ij}(\mu_i^2) \;= 0\,, \qquad (3.2.19)$$

$$K_{5,ij}(\mu_i^2) - \mu_i K_{5\gamma,ij}(\mu_i^2) = 0\,. \qquad (3.2.20)$$

The off-diagonal conditions are only for $i < j$. The off-diagonal conditions for $i > j$ are not independent because of the Hermiticity condition (3.1.12). The condition (3.2.13) imposes a condition on the self-energy but does not impose any condition on the inverse propagator.

Upon substitution of (3.1.19)–(3.1.22) into (3.2.14)–(3.2.16) the on-diagonal renormalization conditions are obtained in terms of the renormalization constants.

$$-\frac{1}{2}\left(\mathbf{Z}_L^{1/2\dagger}\mathbf{M}_0\mathbf{Z}_R^{1/2} + \mathbf{Z}_R^{1/2\dagger}\mathbf{M}_0\mathbf{Z}_L^{1/2}\right)_{ii} + \frac{1}{2}\mu_i\left(\left|\mathbf{Z}_L^{1/2}\right|^2 + \left|\mathbf{Z}_R^{1/2}\right|^2\right)_{ii}$$
$$= -\left[\Sigma_{1,ii}(\mu_i^2) + \mu_i\Sigma_{\gamma,ii}(\mu_i^2)\right]\,, \qquad (3.2.21)$$

$$\frac{1}{2}\left(\left|\mathbf{Z}_L^{1/2}\right|^2 - \left|\mathbf{Z}_R^{1/2}\right|^2\right)_{ii} = -\Sigma_{5\gamma,ii}(\mu_i^2)\,, \qquad (3.2.22)$$

$$\frac{1}{2}\left(\left|\mathbf{Z}_L^{1/2}\right|^2 + \left|\mathbf{Z}_R^{1/2}\right|^2\right)_{ii} = 1 - \Sigma_{\gamma,ii}(\mu_i^2) - 2\mu_i\left[\Sigma'_{1,ii}(\mu_i^2) + \mu_i\Sigma'_{\gamma,ii}(\mu_i^2)\right]\,. \qquad (3.2.23)$$

Upon substitution of (3.1.19)–(3.1.22) into (3.2.17)–(3.2.20) the off-diagonal renormalization conditions are obtained in terms of the renormalization constants ($i < j$)

$$-\frac{1}{2}\left(\mathbf{Z}_L^{1/2\dagger}\mathrm{M}_0\mathbf{Z}_R^{1/2}+\mathbf{Z}_R^{1/2\dagger}\mathrm{M}_0\mathbf{Z}_L^{1/2}\right)_{ij}+\frac{1}{2}\mu_j\left(\left|\mathbf{Z}_L^{1/2}\right|^2+\left|\mathbf{Z}_R^{1/2}\right|^2\right)_{ij}$$
$$=-\left[\Sigma_{1,ij}(\mu_j^2)+\mu_j\Sigma_{\gamma,ij}(\mu_j^2)\right] \quad , \qquad (3.2.24)$$

$$-\frac{1}{2}\left(\mathbf{Z}_L^{1/2\dagger}\mathrm{M}_0\mathbf{Z}_R^{1/2}-\mathbf{Z}_R^{1/2\dagger}\mathrm{M}_0\mathbf{Z}_L^{1/2}\right)_{ij}+\frac{1}{2}\mu_j\left(\left|\mathbf{Z}_L^{1/2}\right|^2-\left|\mathbf{Z}_R^{1/2}\right|^2\right)_{ij}$$
$$=-\left[\Sigma_{5,ij}(\mu_j^2)+\mu_j\Sigma_{5\gamma,ij}(\mu_j^2)\right] , \qquad (3.2.25)$$

$$-\frac{1}{2}\left(\mathbf{Z}_L^{1/2\dagger}\mathrm{M}_0\mathbf{Z}_R^{1/2}+\mathbf{Z}_R^{1/2\dagger}\mathrm{M}_0\mathbf{Z}_L^{1/2}\right)_{ij}+\frac{1}{2}\mu_i\left(\left|\mathbf{Z}_L^{1/2}\right|^2+\left|\mathbf{Z}_R^{1/2}\right|^2\right)_{ij}$$
$$=-\left[\Sigma_{1,ij}(\mu_i^2)+\mu_i\Sigma_{\gamma,ij}(\mu_i^2)\right] \quad , \qquad (3.2.26)$$

$$-\frac{1}{2}\left(\mathbf{Z}_L^{1/2\dagger}\mathrm{M}_0\mathbf{Z}_R^{1/2}-\mathbf{Z}_R^{1/2\dagger}\mathrm{M}_0\mathbf{Z}_L^{1/2}\right)_{ij}-\frac{1}{2}\mu_i\left(\left|\mathbf{Z}_L^{1/2}\right|^2-\left|\mathbf{Z}_R^{1/2}\right|^2\right)_{ij}$$
$$=-\left[\Sigma_{5,ij}(\mu_i^2)-\mu_i\Sigma_{5\gamma,ij}(\mu_i^2)\right] . \qquad (3.2.27)$$

In addition, there is the condition (3.2.13) on the $\gamma_5\not{p}$ component of the diagonal self-energy:

$$\Sigma'_{5\gamma,ii}(\mu_i^2)=0 . \qquad (3.2.28)$$

There is also the CP-invariance requirement on the wavefunction renormalization condition (3.1.24), and the condition on the γ_5 component of the self-energy (3.1.25).

From (3.1.12) it can be seen that the diagonal elements of the self-energies are real, except for the γ_5 component. Thus, (3.2.21)–(3.2.23) represent $3N$ conditions. Now, $\mathbf{K}(\not{p})$ is $N\times N$ so there are N^2 elements. There are N^2-N off-diagonal elements. There are $(N^2-N)/2$ terms in the lower-triangular part. In (3.2.24)–(3.2.27) there are four equations so there are $2(N^2-N)$ conditions. But each of the four equations is complex. The real and imaginary parts must be satisfied separately so actually there are twice as many conditions. That is, there are actually $4(N^2-N)$ conditions. Therefore, altogether there are $3N+4(N^2-N)=4N^2-N$ conditions.

There are N^2 elements in $\mathbf{Z}_L^{1/2}$. It is assumed that the diagonal elements are real. The N^2-N off-diagonal elements are complex so there are actually $2(N^2-N)$ off-diagonal elements. Altogether, $\mathbf{Z}_L^{1/2}$ contains $2(N^2-N)+N=2N^2-N$ constants. There is the same number of constants in $\mathbf{Z}_R^{1/2}$. The mass counterterms δm_i are real so there are only N of them. Therefore, $\mathbf{Z}_L^{1/2}$, $\mathbf{Z}_R^{1/2}$ and δm_i contain $2(2N^2-N)+N=4N^2-N$ constants.

Therefore, there are the same number of constants as there are equations. This result is not in itself a proof of renormalizability, but it is a necessary condition for renormalizability.

3.3 Linearization and Solution of the Renormalization Equations

In the context of perturbative field theory the renormalization equations must be linearized. Assume that the wavefunction renormalization constants have the following form:

$$\mathbf{Z}_L^{1/2}=\begin{pmatrix} 1+x_{11} & x_{12}+iy_{12} & \dots & x_{1N}+iy_{1N} \\ x_{21}+iy_{21} & 1+x_{22} & \dots & x_{2N}+iy_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N1}+iy_{N1} & x_{N2}+iy_{N2} & \dots & 1+x_{NN} \end{pmatrix}, \qquad (3.3.1)$$

$$\mathbf{Z}_R^{1/2}=\begin{pmatrix} 1+u_{11} & u_{12}+iv_{12} & \dots & u_{1N}+iv_{1N} \\ u_{21}+iv_{21} & 1+u_{22} & \dots & u_{2N}+iv_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ u_{N1}+iv_{N1} & u_{N2}+iv_{N2} & \dots & 1+u_{NN} \end{pmatrix}. \qquad (3.3.2)$$

The bare mass is related to the mass counterterm by

$$m_i^0 = m_i - \delta m_i \,. \tag{3.3.3}$$

All of the constants x, y, u, v and δm are all of order $O(g_W^2)$. In short the renormalization constants have the following form:

$$\begin{aligned} Z_{L,ij}^{1/2} &= \delta_{ij} + x_{ij} + i(1-\delta_{ij})y_{ij} \,, \\ Z_{R,ij}^{1/2} &= \delta_{ij} + u_{ij} + i(1-\delta_{ij})v_{ij} \,. \end{aligned} \tag{3.3.4}$$

With this approximation, one obtains the following formulas for combinations of the renormalization constants:

$$\frac{1}{2}\left(\mathbf{Z}_L^{1/2\,\dagger}\mathbf{M}_0\mathbf{Z}_R^{1/2} + \mathbf{Z}_R^{1/2\,\dagger}\mathbf{M}_0\mathbf{Z}_L^{1/2}\right)_{ii} = m_i(Z_{L,ii}^{1/2} + Z_{R,ii}^{1/2} - 1) - \delta m_i \,, \tag{3.3.5}$$

$$\frac{1}{2}\left(\left|\mathbf{Z}_L^{1/2}\right|^2 + \left|\mathbf{Z}_R^{1/2}\right|^2\right)_{ii} = (Z_{L,ii}^{1/2} + Z_{R,ii}^{1/2} - 1) \,, \tag{3.3.6}$$

$$\frac{1}{2}\left(\left|\mathbf{Z}_L^{1/2}\right|^2 - \left|\mathbf{Z}_R^{1/2}\right|^2\right)_{ii} = (Z_{L,ii}^{1/2} + Z_{R,ii}^{1/2}) \,. \tag{3.3.7}$$

And for off-diagonal combinations, one obtains

$$\begin{aligned} &\frac{1}{2}\left(\mathbf{Z}_L^{1/2\,\dagger}\mathbf{M}_0\mathbf{Z}_R^{1/2} + \mathbf{Z}_R^{1/2\,\dagger}\mathbf{M}_0\mathbf{Z}_L^{1/2}\right)_{ij} \\ &\qquad = \frac{1}{2}m_i\left[Z_{L,ij}^{1/2} + Z_{R,ij}^{1/2}\right] + \frac{1}{2}m_j\left[Z_{L,ij}^{1/2\,\dagger} + Z_{R,ij}^{1/2\,\dagger}\right] \,, \end{aligned} \tag{3.3.8}$$

$$\begin{aligned} &\frac{1}{2}\left(\mathbf{Z}_L^{1/2\,\dagger}\mathbf{M}_0\mathbf{Z}_R^{1/2} - \mathbf{Z}_R^{1/2\,\dagger}\mathbf{M}_0\mathbf{Z}_L^{1/2}\right)_{ij} \\ &\qquad = -\frac{1}{2}m_i\left[Z_{L,ij}^{1/2} - Z_{R,ij}^{1/2}\right] + \frac{1}{2}m_j\left[Z_{L,ij}^{1/2} - Z_{R,ij}^{1/2}\right] \,, \end{aligned} \tag{3.3.9}$$

$$\frac{1}{2}\left(\left|\mathbf{Z}_L^{1/2}\right|^2 + \left|\mathbf{Z}_R^{1/2}\right|^2\right)_{ij} = \frac{1}{2}\left[Z_{L,ij}^{1/2} + Z_{R,ij}^{1/2}\right] + \frac{1}{2}\left[Z_{L,ij}^{1/2\,\dagger} + Z_{R,ij}^{1/2\,\dagger}\right] \,, \tag{3.3.10}$$

$$\frac{1}{2}\left(\left|\mathbf{Z}_L^{1/2}\right|^2 - \left|\mathbf{Z}_R^{1/2}\right|^2\right)_{ij} = \frac{1}{2}\left[Z_{L,ij}^{1/2} - Z_{R,ij}^{1/2}\right] + \frac{1}{2}\left[Z_{L,ij}^{1/2} - Z_{R,ij}^{1/2}\right] \,. \tag{3.3.11}$$

Using these combinations the equations (3.2.21)–(3.2.27) can be solved for the renormalization constants. For the on–diagonal constants, one obtains

$$\delta m_i + (\mu_i - m_i) = -\left[\Sigma_{1,ii}(\mu_i^2) + \mu_i \Sigma_{\gamma,ii}(\mu_i^2)\right], \tag{3.3.12}$$

$$Z^{1/2}_{L,ii} = 1 - \frac{1}{2}\Sigma_{\gamma,ii}(\mu_i^2) - \frac{1}{2}\Sigma_{5\gamma,ii}(\mu_i^2) - \mu_i\left[\Sigma'_{1,ii}(\mu_i^2) + \mu_i \Sigma'_{\gamma,ii}(\mu_i^2)\right], \tag{3.3.13}$$

$$Z^{1/2}_{R,ii} = 1 - \frac{1}{2}\Sigma_{\gamma,ii}(\mu_i^2) + \frac{1}{2}\Sigma_{5\gamma,ii}(\mu_i^2) - \mu_i\left[\Sigma'_{1,ii}(\mu_i^2) + \mu_i \Sigma'_{\gamma,ii}(\mu_i^2)\right], \tag{3.3.14}$$

and for the off–diagonal constants ($i < j$),

$$Z^{1/2}_{L,ij} = \frac{\left[\Sigma_{1,ij}(\mu_j^2) + \mu_j \Sigma_{\gamma,ij}(\mu_j^2)\right]}{(m_i - \mu_j)} - \frac{\left[\Sigma_{5,ij}(\mu_j^2) + \mu_j \Sigma_{5\gamma,ij}(\mu_j^2)\right]}{(m_i + \mu_j)}, \tag{3.3.15}$$

$$Z^{1/2}_{R,ij} = \frac{\left[\Sigma_{1,ij}(\mu_j^2) + \mu_j \Sigma_{\gamma,ij}(\mu_j^2)\right]}{(m_i - \mu_j)} + \frac{\left[\Sigma_{5,ij}(\mu_j^2) + \mu_j \Sigma_{5\gamma,ij}(\mu_j^2)\right]}{(m_i + \mu_j)}, \tag{3.3.16}$$

$$Z^{1/2\dagger}_{L,ij} = -\frac{\left[\Sigma_{1,ij}(\mu_i^2) + \mu_i \Sigma_{\gamma,ij}(\mu_i^2)\right]}{(\mu_i - m_j)} + \frac{\left[\Sigma_{5,ij}(\mu_i^2) - \mu_i \Sigma_{5\gamma,ij}(\mu_i^2)\right]}{(\mu_i + m_j)}, \tag{3.3.17}$$

$$Z^{1/2\dagger}_{R,ij} = -\frac{\left[\Sigma_{1,ij}(\mu_i^2) + \mu_i \Sigma_{\gamma,ij}(\mu_i^2)\right]}{(\mu_i - m_j)} - \frac{\left[\Sigma_{5,ij}(\mu_i^2) - \mu_i \Sigma_{5\gamma,ij}(\mu_i^2)\right]}{(\mu_i + m_j)}. \tag{3.3.18}$$

In addition, (3.2.28) is a condition on the $\gamma_5 \not{p}$ component of the self–energy. Also, the CP–invariance conditions (3.1.24) and (3.1.25) must be kept in mind.

CHAPTER FOUR
THE MASS RENORMALIZATION EQUATION

4.1 The Current and Constituent Mass

The mass that appears in the Lagrangian is generated by spontaneous symmetry breaking. This mass, m, is not directly observable because the strong interaction confines quarks to bound states; nevertheless, with the use of the PCAC hypothesis,[25] one can write the current masses in terms of observable hadronic masses;[1,26] for instance,[27]

$$\frac{m_d}{m_u} \approx \frac{m^2(K^0) - m^2(K^+) + m^2(\pi^+)}{2m^2(\pi^0) + m^2(K^+) - m^2(K^0) - m^2(\pi^+)} = 1.80\,, \tag{4.1.1}$$

$$\frac{m_s}{m_d} \approx \frac{m^2(K^0) + m^2(K^+) - m^2(\pi^+)}{m^2(K^0) - m^2(K^+) + m^2(\pi^+)} = 20.1\,. \tag{4.1.2}$$

The ratios are consistent with the values

$$\begin{aligned} m_u &= 4.2 \text{ MeV} \\ m_d &= 7.5 \text{ MeV} \\ m_s &= 150 \text{ MeV} \end{aligned} \tag{4.1.3}$$

Because the mass relationships are derived with the use of current algebra, the Lagrangian mass is often referr d to as the current mass. There is some justification for the masses given in equation (4.1.3), particularly in approaches to hadron phenomenology based on chiral perturbation theory.[1,28]

If the Lagrangian mass cannot be observed directly, then what is the effective mass of the quarks?

Because QCD is responsible for confining quarks, one expects QCD to generate the difference between the current mass and the mass that characterizes static hadronic properties; the effective mass of the quark bound inside hadrons.[29] This mass includes the effects of the gluons that are responsible for binding the quarks, and is, roughly speaking, a fraction of the hadron mass.

As an example, consider the proton, which has a mass of approximately 940 MeV. The proton is made up of two u–quarks and one d–quark. If the u–and d–quarks have the same mass, then $\mu_{u,d} \approx m_{proton}/3$, i.e., $\mu_{u,d} \approx 313$ MeV. Baryon spectroscopy and magnetic moments are modelled well in a static quark model if one uses quark masses between 330 and 360 MeV.[1,30,31] These masses, obtained by regarding the quarks as massive constituents of hadrons, are referred to as the constituent quark masses.

PCAC predicts that the current masses m_u and m_d differ slightly. So one would expect that the constituent masses μ_u and μ_d would also differ slightly. This is in fact born out by the proton–neutron mass difference, which can be attributed to the u–d mass difference. In addition, the PCAC hypothesis predicts that the s–quark will be considerably heavier than both the u–and d–quarks. Hadronic spectroscopy similarly indicates that the constituent mass of the s–quark will be larger than the constituent mass of the u–and d–quarks. The Λ particle, which has a mass of about 1115 MeV, contains one u–quark, one d–quark and one s–quark. Using the value 313 MeV for the u–and d–quarks, one crudely estimates an s–quark constituent mass of about 489 MeV. If one considers the Ω^-, which has a mass of 1672 MeV and contains three s–quarks, then one obtains $\mu_s \approx m_{\Omega^-}/3 = 557 MeV$.

Therefore, the constituent masses are of the order of 300 MeV for the u–and d–quark and 500 MeV for the s–quark. Such masses are justified phenomenologically by considering hadron spectroscopy, as in the previous paragraph, and nucleon magnetic moments.

The question remains as to whether a constituent mass μ that is vastly larger than the Lagrangian mass m can be derived from theoretical considerations.

Because of problems with divergences, any perturbative calculation requires renormalization. The renormalization procedure redefines the bare masses, coupling constants and fields that appear in the Lagrangian in terms of renormalization constants and renormalized quantities. For example, if m^0, g^0 and $\phi^0(x)$ are the bare mass, coupling and field, then one defines $m^0 = m - \delta m$, $g^0 = Z_g g$ and $\phi^0(x) = Z^{1/2}\phi(x)$. The quantities δm, Z_g and $Z^{1/2}$ are the renormalization constants, and m, g and $\phi(x)$ are the renormalized quantities associated with observable physical quantities through suitably chosen renormalization conditions. The renormalization conditions are imposed on Green's functions at a particular value of the momentum called the renormalization point. This procedure is analogous to the imposition of boundary conditions in order to determine the constants of integration in the solution of a differential equation. There are many renormalization schemes in the literature, although different schemes must ultimately lead to the same physics.

The scheme that is central to this thesis is on–mass–shell renormalization. In this particular scheme the renormalization point is identified with the constituent mass.

As has already been mentioned, the constituent mass rather than the Lagrangian mass appears to characterize the properties of hadrons at low momentum. Therefore, the renormalization scheme is performed on a constituent mass–shell *differing* from the Lagrangian mass. This results in a relation between the quarks current mass and its effective mass inside a hadron.

4.2 The Mass–Counterterm and the Mass–Shift

In this section, a relationship is obtained between the Lagrangian mass and the renormalization subtraction mass. A nonperturbative contribution to the self–energy is introduced in order to identify the mass defined by the renormalization scheme with the constituent quark mass.

At the low energies that characterize the hadronic environment of constituent quarks, nonperturbative contributions from the QCD vacuum become significant. If the renormalization point μ is a constituent mass differing from the Lagrangian mass m, then the quark self–energy will contain both a perturbative (P) and a nonperturbative (NP) part:

$$\begin{aligned}\delta m_i + (\mu_i - m_i) = & -\left[\Sigma_{1,ii}(\mu_i^2)^P + \mu_i \Sigma_{\gamma,ii}(\mu_i^2)^P\right] \\ & - \left[\Sigma_{1,ii}(\mu_i^2)^{NP} + \mu_i \Sigma_{\gamma,ii}(\mu_i^2)^{NP}\right] . \quad (4.2.1)\end{aligned}$$

The nonperturbative part arising from QCD vacuum condensates is finite to leading order.[8,9] The mass counterterm is assumed to absorb the entire perturbative part

$$\delta m_i = -\left[\Sigma_{1,ii}(\mu_i^2)^P + \mu_i \Sigma_{\gamma,ii}(\mu_i^2)^P\right], \tag{4.2.2}$$

consistent with the complete renormalization of electroweak amplitudes *prior to* the consideration of vacuum–condensate effects. In this case, from equation (4.2.1) the mass shift is

$$\mu_i - m_i = -\left[\Sigma_{1,ii}(\mu_i^2)^{NP} + \mu_i \Sigma_{\gamma,ii}(\mu_i^2)^{NP}\right]. \tag{4.2.3}$$

The difference between the current and constituent mass is assumed to arise entirely from the nonperturbative part of the self-energy.

The nonperturbative contributions to the constituent mass arise, at least in part, from the QCD and electroweak coupling of the quark–antiquark condensate to the quark self-energy, which are denoted below as "strong" and "weak" respectively:

$$\begin{aligned} \mu_i - m_i = &-\left[\Sigma_{1,ii}(\mu_i^2)^{NP}_{\text{strong}} + \mu_i \Sigma_{\gamma,ii}(\mu_i^2)^{NP}_{\text{strong}}\right] \\ &- \left[\Sigma_{1,ii}(\mu_i^2)^{NP}_{\text{weak}} + \mu_i \Sigma_{\gamma,ii}(\mu_i^2)^{NP}_{\text{weak}}\right]. \end{aligned} \tag{4.2.4}$$

To see if this equation can actually produce a value for μ consistent with the constituent mass one must compute these strong- and electroweak-mediated portions of the nonperturbative self-energy. In Chapter 5, the QCD contribution to the mass shift is discussed. In Chapter 6, the electroweak contribution is computed and added to the QCD contribution.

CHAPTER FIVE
THE QCD–INTERACTION CONTRIBUTION TO THE NONPERTURBATIVE QUARK MASS SHIFT

5.1 The QCD–mediated Nonperturbative Self–Energy

Before the QCD–mediated nonperturbative self–energy is calculated, a brief review[32,33] of how it arises is given. To begin, let $\mid k_1 \cdots k_m \; k'_1 \cdots k'_{m'} \; A \text{ in} \,\rangle$ denote the initial state of a collection of m noninteracting particles and m' noninteracting antiparticles. The quantities $k_1, \ldots, k_m$ and $k'_1, \ldots, k'_{m'}$ are the momenta of the particles and antiparticles respectively and the label A represents all of the quantum numbers of the incoming particles including their spin. Similarly, let $\mid q_1 \cdots q_n \; q'_1 \cdots q'_{n'} \; B \text{ out} \,\rangle$ denote the final state of n outgoing particles and n' outgoing antiparticles. The probability amplitude for the initial state to end up in the final state is just the inner product

$$S_{BA} = \langle \text{ out } q_1 \cdots q_n \; q'_1 \cdots q'_{n'} \; B \mid k_1 \cdots k_m \; k'_1 \cdots k'_{m'} \; A \text{ in} \,\rangle \,. \tag{5.1.1}$$

This amplitude is an element of the S–matrix. Now the reduction formalism developed by Lehmann, Symanzik and Zimmermann[34] (LSZ) can be applied. This formalism allows one to write the S–matrix element in terms of the chronological product of the Heisenberg fields. In order to use the LSZ formalism, one must use the creation and annihilation operators that are defined by the field of an incoming Dirac field,

$$\psi_{\text{in}}(x) = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \frac{m}{k^0} \sum_{\epsilon=\pm 1} \left[b_{\text{in}}(k,\epsilon) u(k,\epsilon) e^{-ik\cdot x} + d^{\dagger}_{\text{in}}(k,\epsilon) v(k,\epsilon) e^{ik\cdot x} \right] . \tag{5.1.2}$$

Under application of second quantization, $b_{\text{in}}^{\dagger}(k,\epsilon)$ and $b_{\text{in}}(k,\epsilon)$ are the creation and annihilation operators for positive–energy particles and $d_{\text{in}}^{\dagger}(k,\epsilon)$ and $d_{\text{in}}(k,\epsilon)$ are the creation and annihilation operators for negative–energy particles (positive–energy antiparticles). Similar assignments apply to the outgoing operators. If the spin is ignored, then the S–matrix element (5.1.1) can be rewritten as

$$\begin{aligned}
&\langle \text{ out } q_1 \cdots q_n \; q_1' \cdots q_{n'}' \; B \mid k_1 \cdots k_m \; k_1' \cdots k_{m'}' \; A \text{ in } \rangle \\
&= \langle \text{ out } B \, | d_{\text{out}}(q_1') \cdots d_{\text{out}}(q_{n'}') b_{\text{out}}(q_1) \cdots b_{\text{out}}(q_n) \\
&\qquad \times b_{\text{in}}^{\dagger}(k_1) \cdots b_{\text{in}}^{\dagger}(k_m) d_{\text{in}}^{\dagger}(k_1') \cdots d_{\text{in}}^{\dagger}(k_{m'}') | \; A \text{ in } \rangle . \qquad (5.1.3)
\end{aligned}$$

Now the reduction formula can be used to obtain

$$\begin{aligned}
S_{BA} &= \left(\frac{-i}{\sqrt{Z_2}}\right)^{m+n} \left(\frac{i}{\sqrt{Z_2}}\right)^{m'+n'} \\
&\times \int \mathrm{d}^4x_1 \cdots \mathrm{d}^4x_m \mathrm{d}^4x_1' \cdots \mathrm{d}^4x_{m'}' \mathrm{d}^4y_1 \cdots \mathrm{d}^4y_n \mathrm{d}^4y_1' \cdots \mathrm{d}^4y_{n'}' \\
&\times \exp\left[-i\left(\sum_{j=1}^{m} k_j \cdot x_j + \sum_{j=1}^{m'} k_j' \cdot x_j' - \sum_{j=1}^{n} q_j \cdot y_j - \sum_{j=1}^{n'} q_j' \cdot y_j'\right)\right] \\
&\times \bar{u}(q_1)(i\overrightarrow{\not\partial}_{y_1} - m) \cdots \bar{u}(q_n)(i\overrightarrow{\not\partial}_{y_n} - m)\bar{v}(k_1')(i\overrightarrow{\not\partial}_{x_1'} - m) \cdots \bar{v}(k_{m'}')(i\overrightarrow{\not\partial}_{x_{m'}'} - m) \\
&\times \langle \tilde{0} | T \left[\overline{\psi}(y_1') \cdots \overline{\psi}(y_{n'}') \psi(y_1) \cdots \psi(y_n) \overline{\psi}(x_1) \cdots \overline{\psi}(x_m) \psi(x_1') \cdots \psi(x_{m'}') \right] | \tilde{0} \rangle \\
&\times (-i\overleftarrow{\not\partial}_{x_1} - m) u(k_1) \cdots (-i\overleftarrow{\not\partial}_{x_m} - m) u(k_m) \\
&\qquad \times (-i\overleftarrow{\not\partial}_{y_1'} - m) v(q_1') \cdots (-i\overleftarrow{\not\partial}_{y_{n'}'} - m) v(q_{n'}') , \qquad (5.1.4)
\end{aligned}$$

where x and x' are the space–time variables conjugate to the incoming particles and antiparticles respectively, and y and y' are the space–time variables conjugate to the outgoing particles and antiparticles respectively. The problem of computing the S-matrix element becomes the problem of computing the chronological product of Heisenberg fields. This product is referred to as the Green's function. In order to simplify the discussion, only the Green's function for one incoming and one outgoing particle is considered. In this case, equation (5.1.4) reduces to

$$S_{BA} = \left(\frac{-i}{\sqrt{Z_2}}\right)^2 \int \mathrm{d}^4x\,\mathrm{d}^4y\, \exp\left[-i(k\cdot x - q\cdot y)\right] \bar{u}(q)(i\overrightarrow{\not\partial}_y - m) \\ \times \langle\tilde{0}|T\left[\psi(y)\overline{\psi}(x)\right]|\tilde{0}\rangle(-i\overleftarrow{\not\partial}_x - m)u(k). \quad (5.1.5)$$

The Green's function is

$$G(y,x) = \langle\tilde{0}|T\left[\psi(y)\overline{\psi}(x)\right]|\tilde{0}\rangle. \quad (5.1.6)$$

To compute the Green function, one assumes that the Heisenberg fields are related to the incoming fields by a unitary transformation

$$\begin{aligned} \psi(x) &= U^{-1}(t)\psi_{\text{in}}(x)U(t), \\ \overline{\psi}(x) &= U^{-1}(t)\overline{\psi}_{\text{in}}(x)U(t). \end{aligned} \quad (5.1.7)$$

The unitary transformation is related to the interaction part of the Hamiltonian $H_I(t)$ by

$$U(t)U^{-1}(t') \equiv U(t,t') = T\left(\exp\left[-i\int_{t'}^{t} \mathrm{d}t'' H_I(t'')\right]\right), \quad (5.1.8)$$

where $H_I(t)$ is written in terms of the incoming fields. Substituting (5.1.7) and (5.1.8) into the Green's function (5.1.6), one obtains

$$\langle\tilde{0}|T\left[\psi(y)\overline{\psi}(x)\right]|\tilde{0}\rangle \\ = \langle\tilde{0}|U^{-1}(t)T\left(\psi_{\text{in}}(y)\overline{\psi}_{\text{in}}(x)\exp\left[-i\int_{t'}^{t} \mathrm{d}t'' H_I(t'')\right]\right)U(-t)|\tilde{0}\rangle. \quad (5.1.9)$$

The vacuum $|\tilde{0}\rangle$ is the ground state of the full Hamiltonian, i.e., it satisfies $H|\tilde{0}\rangle = 0$. The ground state is assumed to be an eigenvector of the unitary operator

$$\langle\tilde{0}|T\left[\psi(y)\overline{\psi}(x)\right]|\tilde{0}\rangle = \frac{\langle\tilde{0}|T\Big(\psi_{\rm in}(y)\overline{\psi}_{\rm in}(x)\exp\left[-i\int_{t'}^{t}{\rm d}t''H_I(t'')\right]\Big)|\tilde{0}\rangle}{\langle\tilde{0}|T\Big(\exp\left[-i\int_{t'}^{t}{\rm d}t''H_I(t'')\right]\Big)|\tilde{0}\rangle}. \qquad (5.1.10)$$

Take the limit as $t\to\infty$, write the Hamiltonian in terms of the Hamiltonian density $\int_{-\infty}^{\infty}{\rm d}tH_I(t) = \int {\rm d}^4z\,\mathcal{H}_I(z)$, and expand the exponential:

$$\langle\tilde{0}|T\left[\psi(y)\overline{\psi}(x)\right]|\tilde{0}\rangle = \frac{\sum_{m=0}^{\infty}\frac{(-i)^m}{m!}\int {\rm d}^4z_1\cdots{\rm d}^4z_m\,\langle\tilde{0}|T\Big(\psi_{\rm in}(y)\overline{\psi}_{\rm in}(x)\mathcal{H}_I(z_1)\cdots\mathcal{H}_I(z_m)\Big)|\tilde{0}\rangle}{\sum_{m=0}^{\infty}\frac{(-i)^m}{m!}\int {\rm d}^4z_1\cdots{\rm d}^4z_m\,\langle\tilde{0}|T\Big(\mathcal{H}_I(z_1)\cdots\mathcal{H}_I(z_m)\Big)|\tilde{0}\rangle}. \qquad (5.1.11)$$

As an example, consider the QCD interaction Hamiltonian, which has the form

$$\mathcal{H}_{\rm in}(z) = g_s\sum_{\alpha}\sum_{\rho,\sigma,a}\overline{\psi}^{\rho}_{{\rm in},\alpha}(z)\gamma_\mu t^a_{\rho\sigma}\psi^{\sigma}_{{\rm in},\alpha}(z)B^{\mu}_{{\rm in},a}(z), \qquad (5.1.12)$$

where α is a flavour index, $a = 1,2,\ldots,8$ are the indices for the adjoint representation of $SU_C(3)$, and $\rho,\sigma = r,y,b$ are colour indices (red, yellow, blue). In order to simplify the discussion, the interacting Hamiltonian is written in a generalized matrix notation,

$$\mathcal{H}_{\rm in}(z) = g_s\overline{\psi}_{\rm in}(z)B_{\rm in}(z)\psi_{\rm in}(z), \qquad (5.1.13)$$

where $B(z) = \gamma_\mu t^a B^\mu_a$. With this form, the Green's function (5.1.11) contains a term of $O(g_s^2)$ of the form

$$iS_F^{(2)}(\not{p}) = -\frac{g_s^2}{2}\int {\rm d}^4(y-x)\,e^{ip\cdot(y-x)}\int {\rm d}^4z_1{\rm d}^4z_2$$
$$\times\langle\tilde{0}|T\left[\psi_{\rm in}(y)\overline{\psi}_{\rm in}(x)\overline{\psi}_{\rm in}(z_1)B_{\rm in}(z_1)\psi_{\rm in}(z_1)\overline{\psi}_{\rm in}(z_2)B_{\rm in}(z_2)\psi_{\rm in}(z_2)\right]|\tilde{0}\rangle. \qquad (5.1.14)$$

The denominator of (5.2.11) has been ignored, it being understood that only the connected part is to be considered. The Green's function has been transformed to momentum space:

$$iS_F(\not{p}) = \int d^4(y-x)\, e^{ip\cdot(y-x)} G(y,x)\,. \tag{5.1.15}$$

Now Wick's Theorem can be applied. For two fermion fields the theorem states that

$$T\left[\psi_{\rm in}(y)\overline{\psi}_{\rm in}(x)\right] = \langle 0|T\left[\psi_{\rm in}(y)\overline{\psi}_{\rm in}(x)\right]|0\rangle + :\psi_{\rm in}(y)\overline{\psi}_{\rm in}(x):\ . \tag{5.1.16}$$

The perturbative vacuum state $|0\rangle$ appearing on the right hand side is annihilated by the annihilation operators that appear in the incoming fields. Nonperturbative content enters the theory by assuming that the vacuum state $|0\rangle$ is distinct from the ground state $|\tilde{0}\rangle$. Thus, the QCD ground state is *not* annihilated by the annihilation operators:

$$\langle\tilde{0}|T\left[\psi_{\rm in}(y)\overline{\psi}_{\rm in}(x)\right]|\tilde{0}\rangle = \langle 0|T\left[\psi_{\rm in}(y)\overline{\psi}_{\rm in}(x)\right]|0\rangle + \langle\tilde{0}|:\psi_{\rm in}(y)\overline{\psi}_{\rm in}(x):|\tilde{0}\rangle\,. \tag{5.1.17}$$

Taking this into account and applying Wick's Theorem to the chronological product in the right hand side of (5.1.14) one obtains two types of terms. The first term is the contribution from the vacuum state $|0\rangle$ only

$$\begin{aligned} i\left[S_F^{(2)}(\not{p})^P\right]_{kl}^{\rho\sigma} &= -\frac{g_s^2}{4}\int d^4(y-x)\, e^{ip\cdot(y-x)}\int d^4z_1\, d^4z_2 \langle 0|T\left[\psi^\rho_{{\rm in},k}(y)\overline{\psi}^\tau_{{\rm in},i}(z_1)\right]|0\rangle \\ &\quad\times\gamma^\mu_{im}\lambda^b_{\tau\alpha}\langle 0|T\left[\psi^\alpha_m(z_1)\overline{\psi}^\beta_{{\rm in},r}(z_2)\right]|0\rangle\gamma^\nu_{rj}\lambda^c_{\beta\omega}\langle 0|T\left[\psi^\omega_j(z_2)\overline{\psi}^\sigma_l(x)\right]|0\rangle \\ &\quad\times\langle 0|T\left[B^b_\mu(z_1)B^c_\nu(z_2)\right]|0\rangle\,, \end{aligned} \tag{5.1.18}$$

where ρ, σ are colour indices, and k, l are Dirac indices. Equation (5.1.18) is the purely perturbative contribution to the propagator. The second term arises because the ground state $|\tilde{0}\rangle$ is not annihilated by the annihilation operators. This term is identified as the nonperturbative contribution

$$
\begin{aligned}
i\left[S_F^{(2)}(\not{p})^{NP}\right]_{kl}^{\rho\sigma}
&= -\frac{g_s^2}{4}\int \mathrm{d}^4(y-x)\, e^{ip\cdot(y-x)} \int \mathrm{d}^4z_1\, \mathrm{d}^4z_2 \langle 0|T\left[\psi^{\rho}_{\mathrm{in},k}(y)\overline{\psi}^{\tau}_{\mathrm{in},i}(z_1)\right]|0\rangle \\
&\qquad \times \gamma^{\mu}_{im}\lambda^{b}_{\tau\alpha}\langle\tilde{0}|{:}\psi^{\alpha}_{m}(z_1)\overline{\psi}^{\beta}_{\mathrm{in},r}(z_2){:}|\tilde{0}\rangle\gamma^{\nu}_{rj}\lambda^{c}_{\beta\omega}\langle 0|T\left[\psi^{\omega}_{j}(z_2)\overline{\psi}^{\sigma}_{l}(x)\right]|0\rangle \\
&\qquad \times \langle 0|T\left[B^{b}_{\mu}(z_1)B^{c}_{\nu}(z_2)\right]|0\rangle\,. \qquad (5.1.19)
\end{aligned}
$$

There are additional nonperturbative contributions arising from higher-dimensional condensates[9], but only the lowest-dimensional contribution (5.1.19) is considered here. The nonzero ground state expectation value of the normal ordered product that appears in (5.1.19) is related to the quark condensate as will be shown in the next section. To reiterate, the quark condensate appears in the perturbative calculation of the propagator because the ground state of the Hamiltonian $|\tilde{0}\rangle$ is not annihilated by the annihilation operators that appear in the incoming fields.

5.2 The Quark Condensate Contribution to the QCD Self-Energy

In the previous section, it was shown how the nonzero ground state expectation value of the normal ordered product arises in the calculation of the quark propagator. In this section, it will be shown how this leads to the appearance of the quark condensate in the quark self-energy. It will also be shown that if the weak contribution is ignored then the renormalization prescription given in Chapter 2 is consistent for QCD alone.

The propagator and the self-energy are related by the expression

$$S_F(\not p) = \frac{1}{\not p - m + \Sigma(\not p)} = \frac{1}{\not p - m} + \frac{1}{\not p - m}\left[-\Sigma(\not p)\right]\frac{1}{\not p - m} + \cdots . \tag{5.2.1}$$

This can be compared to

$$S_F(\not p) = S_F^{(0)}(\not p) + S_F^{(2)}(\not p) + \cdots , \tag{5.2.2}$$

to obtain the relation

$$-\Sigma(\not p) = (\not p - m)S_F^{(2)}(\not p)(\not p - m) . \tag{5.2.3}$$

In the previous section it was shown that the second order term of the propagator $S_F^{(2)}(\not p)$ contains both a perturbative and a nonperturbative part. Thus the self-energy will also contain a perturbative and a nonperturbative part.

$$\Sigma(\not p) = \Sigma(\not p)^{P}_{\text{strong}} + \Sigma(\not p)^{NP}_{\text{strong}} . \tag{5.2.4}$$

The relationship between the mass difference and the nonperturbative self-energy is (4.2.4)

$$\mu_i - m_i = -\left[\Sigma_{1,ii}(\mu_i^2)^{NP}_{\text{strong}} + \mu_i \Sigma_{\gamma,ii}(\mu_i^2)^{NP}_{\text{strong}}\right] , \tag{5.2.5}$$

where the contribution of the weak interaction is ignored.

The part of the QCD self–energy sensitive to the quark condensate is

$$\Sigma^{\rho\sigma}_{\ell n}(\not p)^{NP}_{\mathrm{strong}} = -\frac{g_s^2}{4(2\pi)^4}\lambda^b_{\rho\alpha}\lambda^b_{\beta\sigma}\gamma^\mu_{\ell n}\int \mathrm{d}^4(y-z)e^{i(p-k)\cdot(y-z)}$$
$$\times\int\frac{\mathrm{d}^4k}{(k^2)^2}\langle\tilde{0}|:\psi^\alpha_n(y)\bar{\psi}^\beta_r(z)|\tilde{0}\rangle:\left[-g_{\mu\nu}k^2+(1-a)k_\mu k_\nu\right]\gamma^\nu_{ru}\,. \quad (5.2.6)$$

The perturbative contribution is identical to (5.2.6) except that the normal–ordered product is replaced by the time–ordered product. The ground state expectation value of the normal–ordered product of fermion fields can be evaluated with the use of the operator–product expansion (OPE)[9]

$$\langle\tilde{0}|:\psi^\alpha_n(y)\overline{\psi}^\beta_r(z):|\tilde{0}\rangle = -\delta^{\alpha\beta}\langle\overline{q}q\rangle\sum_{j=0}^{\infty}C_j(-i\tilde{\mu})^j\left[\gamma\cdot(y-z)\right]^j_{nr}$$
$$+\ \text{higher dimensional condensates}\,, \quad (5.2.7)$$

where C_j is defined in (C.2). The mass $\tilde{\mu}$ characterizes the OPE. It arises because the covariant QCD equation of motion

$$\not D\psi(x) = -i\tilde{\mu}\psi(x) \quad (5.2.8)$$

is used in the evaluation of the expansion (5.2.7). Upon substitution of (5.2.7) into (5.2.6) and evaluation of (5.2.6) as described in References 9 and 34, the dimension–3 quark condensate contribution to the nonperturbative self–energy is found to be[8,9,10]

$$\Sigma_{1,ii}(p^2)^{NP}_{\mathrm{strong}} = -g_s^2\frac{(3+a)\langle\overline{\psi}_i\psi_i\rangle}{9p^2} \qquad \Sigma_{\gamma,ii}(p^2)^{NP}_{\mathrm{strong}} = g_s^2\frac{a\tilde{\mu}_i\langle\overline{\psi}_i\psi_i\rangle}{9p^4}\,, \quad (5.2.9)$$

The contribution of higher dimensional condensates, such as $\langle \alpha_s G^2 \rangle$, $|\langle \bar{q}q \rangle|^2$ and so on, are ignored. The mass $\tilde{\mu}_i$, which is the mass that characterizes the nonperturbative fermion propagator[11] $\langle \tilde{0} | : \psi_i(x)\overline{\psi}_i(0) : | \tilde{0} \rangle$, is, at present, left arbitrary. Substituting (5.2.9) into (5.2.5), one finds that

$$\mu_i - m_i = g_s^2 \frac{(3+a)\langle \overline{\psi}_i \psi_i \rangle}{9\mu_i^2} - g_s^2 \mu_i \frac{a\tilde{\mu}_i \langle \overline{\psi}_i \psi_i \rangle}{9\mu_i^4} \,. \tag{5.2.10}$$

If the mass $\tilde{\mu}_i$ is identified with the mass μ_i, then this expression is independent of the gauge parameter a.[8] Setting $\tilde{\mu}_i = \mu_i$, one obtains

$$\mu_i = m_i + g_s^2 \frac{\langle \overline{\psi}_i \psi_i \rangle}{3\mu_i^2} \,. \tag{5.2.11}$$

Thus, gauge invariance of the mass *necessitates* a shift from the purely perturbative mass shell at m_i to the mass–shell at μ_i. The difference between these two masses is determined by the vacuum condensate.

It should be noted that these results are consistent with the dynamical mass function obtained from the integral equations for QCD Green's functions even when they are improved by the use of the renormalization group.[7] The renormalization group in the Landau gauge ($a = 0$) amounts to the substitution of the following quantities into equation (5.2.11):

$$g_s^2 \longrightarrow 4\pi\alpha_s(p^2) = \frac{2}{b \ln(p^2/\Lambda^2)} \,, \tag{5.2.12}$$

$$\langle \overline{\psi}_i \psi_i \rangle \longrightarrow \langle \overline{\psi}_i \psi_i \rangle_p = \langle \overline{\psi}_i \psi_i \rangle_\tau \left[\frac{\ln(p^2/\Lambda^2)}{\ln(\tau^2/\Lambda^2)} \right]^{1/2b\pi^2} \,, \tag{5.2.13}$$

where $b = (33 - 2N)/24\pi^2$ and p^2 is Euclidean ($p^2 < 0$). One then obtains from these improvements to (5.2.11) the dynamical mass function[7]

$$m_{\mathrm{dyn}}(p^2) \equiv \Sigma_1(p^2)^{NP} = C\frac{1}{p^2}\left[\ln(|p^2|/\Lambda^2)\right]^{(1/2b\pi^2)-1}, \tag{5.2.14}$$

which is identical to the dynamical mass function obtained from the Schwinger–Dyson integral equations.[7] The constant C, given by[7]

$$C \equiv \frac{2\langle\overline{\psi}_i\psi_i\rangle_\tau}{3b\left[\ln(\tau^2/\Lambda^2)\right]^{1/2b\pi^2}}. \tag{5.2.15}$$

is invariant under transformations of the renormalization group.[7]

The correspondence between the quark–condensate component of the self–energy mediated by QCD, $\Sigma^1(p^2)$, and the dynamical mass function in the Euclidean momentum regime, $m_{\mathrm{dyn}}(p^2)$, is an important check for the consistency of the renormalization prescription. This correspondence suggests the legitimacy of the identification of (5.2.11) with the expression motivated by phenomenology that relates current (cur), constituent (con) and dynamical (dyn) quark masses[31,36]

$$m_{\mathrm{con}} = m_{\mathrm{cur}} + \frac{m_{\mathrm{dyn}}^3}{m_{\mathrm{con}}^2}. \tag{5.2.16}$$

This identification is made despite the fact that the sign of $\langle\overline{\psi}_i\psi_i\rangle$ for Minkowskian ($p^2 > 0$) momenta is ambiguous.[9] The sign of $\langle\overline{\psi}_i\psi_i\rangle$ must be positive at $p^2 = \mu^2$ to compensate for the change of sign of p^2 in (5.2.10). Such a change of sign is required to identify (5.2.11) with (5.2.16).

Therefore, the QCD–mediated dynamical contribution to the quark self–energy can be treated by the methods presented earlier, leading to equation (5.2.11). The physical applicability of the renormalization prescription employed is shown by the identification of (5.2.11) and (5.2.16).

CHAPTER SIX
ELECTROWEAK–INTERACTION CONTRIBUTIONS TO THE QUARK MASS SHIFT

6.1 The Quark Condensate Contribution to the Electroweak Quark Self–Energy

In this section the nonperturbative, electroweak contributions to the quark self–energies are computed by following the calculation outlined in Sections 5.1 and 5.2 for the case of QCD. The interacting part of the Hamiltonian for the strong interaction (5.1.12) is replaced by the interacting part of the Hamiltonian for the electroweak interaction, which can be found in Section 4.1 of Reference 3. Because the known interactions are described by an $SU(3)_{QCD} \times \left[SU(2)_L \times U(1)\right]_{EW}$ gauge theory, the ground state $|\tilde{0}\rangle$ of the QCD Hamiltonian introduced in Section 5.1 is assumed to be the ground state of the electroweak Hamiltonian as well. That is, it satisfies both $H_{QCD}|\tilde{0}\rangle = 0$ and $H_{EW}|\tilde{0}\rangle = 0$. Once again the ground state $|\tilde{0}\rangle$ is not annihilated by the annihilation operators appearing in the asymptotic fields, regardless of the interaction under consideration. Therefore, the quark propagator will acquire nonperturbative contributions from self–energies mediated by electroweak gauge bosons $W^{\pm}$, Z, their scalar partners $\chi^{\pm}$, χ_3, and the Higgs field ϕ.[3,37] These nonperturbative self energies, like the QCD contribution derived in Chapter 5, contribute to the shift from the renormalized Lagrangian mass, m, to the effective mass, μ.

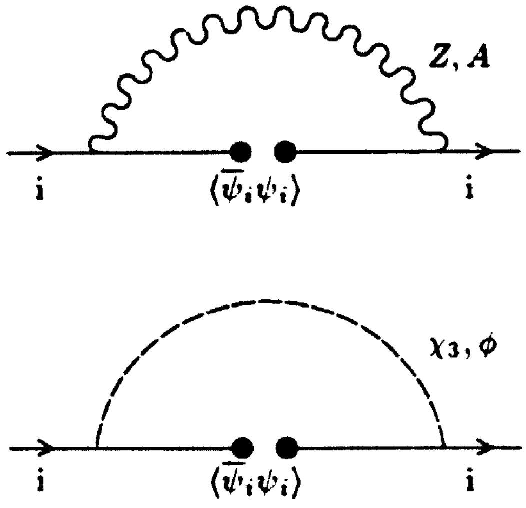


Figure 6.1 Electroweak contributions to flavour–diagonal self–energies of the charge $-1/3$ quarks. χ_3 is the scalar partner to the Z, ϕ is the Higgs field, and χ^+ is the scalar partner to the W^+.

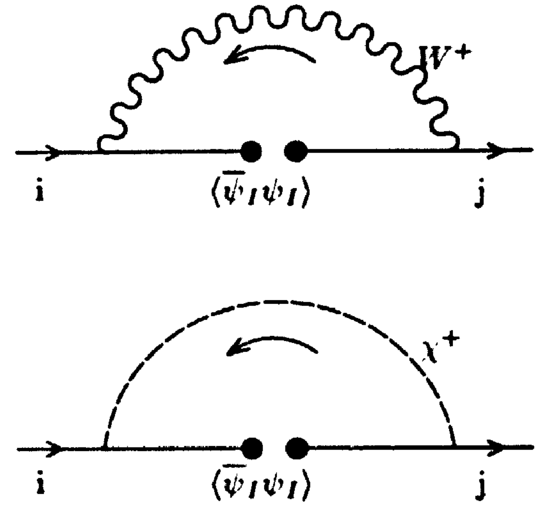


Figure 6.2 Electroweak contributions to flavour–off–diagonal ($i \neq j$) self–energies of charge $-1/3$ quarks. The parameter $\langle\bar\psi_I\psi_I\rangle$ is the condensate of a charge $+2/3$ quark.

Figures 6.1 and 6.2 show the diagrams that represent the quark condensate contributions to the electroweak self–energies of quarks. It is important to note in Figure 6.2 that the exchange of a W^+ boson or its scalar partner χ^+ allows a quark of flavour i to become a quark of flavour j. The flavour j can equal i so W^+ and χ^+ also contribute to flavour–diagonal self–energies. It has already been noted in Chapter 5 that the masses $\tilde\mu_i$ characterizing the nonperturbative ground state expectation value of quark and antiquark fields must necessarily be identified with the masses μ_i. The contributions of the quark condensates to the self–energies that are mediated by the electroweak interactions are

$$-i\Sigma^A_{ij}(\not p)^{NP}_{\mathrm{weak}} = \delta_{ij}\frac{e^2}{9}\gamma^\mu \int \frac{\mathrm{d}^4q}{(2\pi)^4 i}\, S_i(p-q)^{NP} D^A_{\mu\nu}(q)\gamma^\nu\,, \tag{6.1.1}$$

$$-i\Sigma^{Z}_{ij}(\not p)^{NP}_{\text{weak}} = \delta_{ij}\frac{g_W^2 M_Z^2}{16M_W^2}\gamma^\mu(\lambda_i+\gamma_5)$$
$$\times\int\frac{\mathrm{d}^4q}{(2\pi)^4 i}\,S_i(p-q)^{NP}D^{Z}_{\mu\nu}(q)\gamma^\nu(\lambda_i+\gamma_5)\,, \qquad (6.1.2)$$

$$-i\Sigma^{\chi_3}_{ij}(\not p)^{NP}_{\text{weak}} = -\delta_{ij}\frac{g_W^2 m_i^2}{4M_W^2}\gamma_5\int\frac{\mathrm{d}^4q}{(2\pi)^4 i}\,S_i(p-q)^{NP}D^{\chi_3}(q)\gamma_5\,, \qquad (6.1.3)$$

$$-i\Sigma^{\phi}_{ij}(\not p)^{NP}_{\text{weak}} = \delta_{ij}\frac{g_W^2 m_i^2}{4M_W^2}\int\frac{\mathrm{d}^4q}{(2\pi)^4 i}\,S_i(p-q)^{NP}D^{\phi}(q)\,, \qquad (6.1.4)$$

$$-i\Sigma^{W^\pm}_{ij}(\not p)^{NP}_{\text{weak}} = \frac{g_W^2}{8}\sum_{I=1}^{N}(V^\dagger)_{iI}V_{Ij}\,\gamma^\mu(1-\gamma_5)$$
$$\times\int\frac{\mathrm{d}^4q}{(2\pi)^4 i}\,S_I(p-q)^{NP}D^{W}_{\mu\nu}(q)\gamma^\nu(1-\gamma_5)\,, \qquad (6.1.5)$$

$$-i\Sigma^{\chi^\pm}_{ij}(\not p)^{NP}_{\text{weak}} = -\frac{g_W^2}{8M_W^2}\sum_{I=1}^{N}(V^\dagger)_{iI}V_{Ij}\left[(m_I-m_i)+(m_I+m_i)\gamma_5\right]$$
$$\times\int\frac{\mathrm{d}^4q}{(2\pi)^4 i}\,S_I(p-q)^{NP}D^{\chi}(q)\left[(m_j-m_I)+(m_j+m_I)\gamma_5\right]\,, \qquad (6.1.6)$$

where $\lambda_i = -1+4(M_Z^2-M_W^2)/(3M_Z^2)$. Uppercase letters represent charge $+2/3$ quarks, $I=u$, c, and lowercase letters represent charge $-1/3$ quarks, $i=d$, s, b. The top quark is disregarded for reasons discussed below.

The masses of the quarks cannot depend on the arbitrary gauge parameters α_A, α_W and α_Z. The self energies evaluated on the mass-shell contribute to the quark masses according to equation (4.2.4). Therefore, the self-energies must be independent of the arbitrary gauge parameters α_A, α_W and α_Z.

Ahmady *et al.*[37] studied the problem of on-mass-shell gauge parameter independence. They showed that the self-energies are independent of the gauge parameters if the mass-shell μ is identified with the Lagrangian mass m. They further showed that if μ and m are distinct, then one must introduce Ward identity corrections to the vertices in order to maintain gauge parameter independence.[38] However, it is possible to maintain the distinction between μ and m, and to avoid the complications of the Ward identity corrections by restricting the gauge parameters to the Landau gauge $\alpha_A = \alpha_W = \alpha_Z = 0$. Therefore, the self-energies are computed in the Landau gauge.

Using the propagators given in Appendix B and the identity (C.1) , one obtains

$$-i\Sigma_{ij}^{A}(\not{p})_{\text{weak}}^{NP} = \delta_{ij}\frac{e^2}{9}\gamma^{\mu} i\langle\overline{\psi}_i\psi_i\rangle \sum_{k=0}^{\infty} C_k \mu_i^k \left[-\gamma\cdot\frac{\partial}{\partial p}\right]^k \left[\frac{g_{\mu\nu}}{p^2} - \frac{p_\mu p_\nu}{(p^2)^2}\right]\gamma^{\nu}, \quad (6.1.7)$$

$$\begin{aligned}-i\Sigma_{ij}^{Z}(\not{p})_{\text{weak}}^{NP} = {} & \delta_{ij}\frac{g_W^2 M_Z^2}{16M_W^2}\gamma^{\mu}(\lambda_i+\gamma_5) i\langle\overline{\psi}_i\psi_i\rangle \sum_{k=0}^{\infty} C_k \mu_i^k \left[-\gamma\cdot\frac{\partial}{\partial p}\right]^k \\ & \times\left[\frac{g_{\mu\nu}}{p^2-M_Z^2} - \frac{p_\mu p_\nu/M_Z^2}{p^2-M_Z^2} + \frac{p_\mu p_\nu/M_Z^2}{p^2}\right]\gamma^{\nu}(\lambda_i+\gamma_5), \quad (6.1.8)\end{aligned}$$

$$-i\Sigma_{ij}^{\chi_3}(\not{p})_{\text{weak}}^{NP} = -\delta_{ij}\frac{g_W^2 m_i^2}{4M_W^2}\gamma_5 i\langle\overline{\psi}_i\psi_i\rangle \sum_{k=0}^{\infty} C_k \mu_i^k \left[-\gamma\cdot\frac{\partial}{\partial p}\right]^k \frac{1}{p^2}\gamma_5, \quad (6.1.9)$$

$$-i\Sigma_{ij}^{\phi}(\not{p})_{\text{weak}}^{NP} = \delta_{ij}\frac{g_W^2 m_i^2}{4M_W^2} i\langle\overline{\psi}_i\psi_i\rangle \sum_{k=0}^{\infty} C_k \mu_i^k \left[-\gamma\cdot\frac{\partial}{\partial p}\right]^k \frac{1}{m_\phi^2-p^2}, \quad (6.1.10)$$

$$\begin{aligned}-i\Sigma_{ij}^{W^{\pm}}(\not{p})_{\text{weak}}^{NP} = {} & \frac{g_W^2}{8}\sum_{I=1}^{N}(V^{\dagger})_{iI}V_{Ij}\gamma^{\mu}(1-\gamma_5) i\langle\overline{\psi}_I\psi_I\rangle \sum_{k=0}^{\infty} C_k \mu_I^k \left[-\gamma\cdot\frac{\partial}{\partial p}\right]^k \\ & \times\left[\frac{g_{\mu\nu}}{p^2-M_W^2} - \frac{p_\mu p_\nu/M_W^2}{p^2-M_W^2} + \frac{p_\mu p_\nu/M_W^2}{p^2}\right]\gamma^{\nu}(1-\gamma_5), \quad (6.1.11)\end{aligned}$$

$$\begin{aligned}-i\Sigma_{ij}^{\chi^{\pm}}(\not{p})_{\text{weak}}^{NP} = {} & -\frac{g_W^2}{8M_W^2}\sum_{I=1}^{N}(V^{\dagger})_{iI}V_{Ij}\left[(m_I-m_i)+(m_I+m_i)\gamma_5\right] \\ & \times i\langle\overline{\psi}_I\psi_I\rangle \sum_{k=0}^{\infty} C_k \mu_I^k \left[-\gamma\cdot\frac{\partial}{\partial p}\right]^k \frac{1}{p^2}\left[(m_j-m_I)+(m_j+m_I)\gamma_5\right]. \quad (6.1.12)\end{aligned}$$

Now the identities (C.3)-(C.9) are used to arrive at the self-energies

$$\Sigma_{ij}^{A}(\not{p})_{\text{weak}}^{NP} = -\delta_{ij}\frac{e^2\langle\overline{\psi}_i\psi_i\rangle}{36p^2}\mathbf{1}\,, \tag{6.1.13}$$

$$\Sigma_{ij}^{Z}(\not{p})_{\text{weak}}^{NP} = \delta_{ij}\frac{g_W^2\langle\overline{\psi}_i\psi_i\rangle}{384M_W^2}\left\{6(\lambda_i^2-1)\mathbf{1}-(\lambda_i^2+1)\frac{\mu_i(\mu_i^2-4p^2)}{(p^2)^2}\not{p}\right.$$
$$\left.+\,2\lambda_i\frac{\mu_i(\mu_i^2-4p^2)}{(p^2)^2}\gamma_5\not{p}\right\}, \tag{6.1.14}$$

$$\Sigma_{ij}^{\chi_3}(\not{p})_{\text{weak}}^{NP} = -\delta_{ij}\frac{g_W^2m_i^2\langle\overline{\psi}_i\psi_i\rangle}{48M_W^2p^2}\left[\mathbf{1}-\frac{\mu_i}{2p^2}\not{p}\right], \tag{6.1.15}$$

$$\Sigma_{ij}^{\phi}(\not{p})_{\text{weak}}^{NP} = 0\,, \tag{6.1.16}$$

$$\Sigma_{ij}^{W^\pm}(\not{p})_{\text{weak}}^{NP} = -\frac{g_W^2}{96M_W^2(p^2)^2}\sum_{I=1}^{N}(V^\dagger)_{iI}V_{Ij}\,\langle\overline{\psi}_I\psi_I\rangle\mu_I(\mu_I^2-4p^2)(\not{p}+\gamma_5\not{p})\,, \tag{6.1.17}$$

$$\Sigma_{ij}^{\chi^\pm}(\not{p})_{\text{weak}}^{NP} = -\frac{g_W^2}{96M_W^2p^2}\left\{2(m_i+m_j)\sum_{I=1}^{N}(V^\dagger)_{iI}V_{Ij}\,m_I\langle\overline{\psi}_I\psi_I\rangle\mathbf{1}\right.$$
$$-\frac{1}{p^2}\sum_{I=1}^{N}(V^\dagger)_{iI}V_{Ij}\,\mu_I(m_I^2+m_im_j)\langle\overline{\psi}_I\psi_I\rangle\not{p}$$
$$-\,2(m_i-m_j)\sum_{I=1}^{N}(V^\dagger)_{iI}V_{Ij}\,m_I\langle\overline{\psi}_I\psi_I\rangle\gamma_5$$
$$\left.-\frac{1}{p^2}\sum_{I=1}^{N}(V^\dagger)_{iI}V_{Ij}\,\mu_I(m_I^2-m_im_j)\langle\overline{\psi}_I\psi_I\rangle\gamma_5\not{p}\right\}. \tag{6.1.18}$$

The approximations for the series (C.7)-(C.9) ignore all terms of order $O(p^2/M^2)$, $O(m_i^2/M^2)$ and $O(m_I^2/M^2)$. In the above equations, M is either M_W, M_Z or m_ϕ. Consequently, the above equations are valid only for p^2, m_i^2, $m_I^2 << M_W^2$, M_Z^2, m_ϕ. This restriction is upheld by the u, d, c, s and b quarks but is violated by the t–quark, which is assumed to be very heavy. The mass m_t can enter the self–energies of the charge $-1/3$ quarks only by intergenerational mixing involved in $W^{\pm}$ or $\chi^{\pm}$ exchange. In the above equations the contributions from $W^{\pm}$ or $\chi^{\pm}$ exchange correspond to the terms that contain a sum over I. Therefore, because of the restriction, the sums that appear in the self–energy components are valid only for $I = u$, c.

Within the context of the nonperturbative content of the QCD vacuum it may be valid to ignore the t–quark contribution altogether. In the heavy–quark expansion, the t–quark condensate is induced by the gluon condensate,[39,40]

$$\langle \bar{t}t \rangle = -\frac{\langle \alpha_S G^2 \rangle}{12\pi \mu_t} . \qquad (6.1.19)$$

If the t–quark mass is very large, then the t–quark condensate will be very small. Any contribution to the self–energy proportional to $\langle \bar{t}t \rangle$ would be insignificant. The contribution of the t–quark is further reduced by the smallness of the CKM mixing angle.

From equations (6.1.13)–(6.1.18) one collects together the $\mathbf{1}$, $\not{p}$, γ_5 and $\gamma_5 \not{p}$ components of the self–energy:

$$\Sigma_{\mathbf{1},ij}(p^2)^{NP}_{\mathbf{weak}} = -\delta_{ij} \frac{e^2 \langle \bar{\psi}_i \psi_i \rangle}{36 p^2} + \delta_{ij} \frac{g_W^2 \langle \bar{\psi}_i \psi_i \rangle}{192 M_W^2 p^2} \left[3(\lambda_i^2 - 1) p^2 - 4 m_i^2 \right]$$
$$- \frac{g_W^2 (m_i + m_j)}{48 M_W^2 p^2} \sum_{I=1}^{N} (V^{\dagger})_{iI} V_{Ij} m_I \langle \bar{\psi}_I \psi_I \rangle , \qquad (6.1.20)$$

$$\Sigma_{\gamma,ij}(p^2)^{NP}_{\text{weak}} = -\delta_{ij} g_W^2 (\lambda_i^2 + 1) \frac{\mu_i(\mu_i^2 - 4p^2)\langle\overline{\psi}_i\psi_i\rangle}{384 M_W^2 p^4} + \delta_{ij} \frac{g_W^2 m_i^2 \mu_i \langle\overline{\psi}_i\psi_i\rangle}{96 M_W^2 p^4}$$
$$+ \frac{g_W^2}{96 M_W^2 p^4} \sum_{I=1}^{N} (V^\dagger)_{iI} V_{Ij}\, \mu_I (m_I^2 + m_j m_i - \mu_I^2 + 4p^2) \langle\overline{\psi}_I\psi_I\rangle\,, \quad (6.1.21)$$

$$\Sigma_{5,ij}(p^2)^{NP}_{\text{weak}} = -\frac{g_W^2(m_i - m_j)}{48 M_W^2 p^2} \sum_{I=1}^{N} (V^\dagger)_{iI} V_{Ij}\, m_I \langle\overline{\psi}_I\psi_I\rangle\,, \quad (6.1.22)$$

$$\Sigma_{5\gamma,ij}(p^2)^{NP}_{\text{weak}} = \delta_{ij} \frac{g_W^2 \lambda_i \mu_i (\mu_i^2 - 4p^2)\langle\overline{\psi}_i\psi_i\rangle}{192 M_W^2 p^4}$$
$$+ \frac{g_W^2}{96 M_W^2 p^4} \sum_{I=1}^{N} (V^\dagger)_{iI} V_{Ij}\, \mu_I (m_I^2 - m_j m_i - \mu_I^2 + 4p^2) \langle\overline{\psi}_I\psi_I\rangle\,. \quad (6.1.23)$$

In the following section, the **1** and $\not{p}$ components (6.1.20) and (6.1.21) are inserted into the equation for the mass difference (4.2.4)

The self–energies for charge +2/3 quarks are also required. The Feynman diagrams for these self energies are the same as those shown in Figures 6.1 and 6.2 except that the lowercase letters become uppercase letters and *vice versa*. The self–energies are

$$-i\Sigma^A_{IJ}(\not{p})^{NP}_{\text{weak}} = \delta_{IJ} \frac{4e^2}{9} \gamma^\mu \int \frac{\mathrm{d}^4 q}{(2\pi)^4 i}\, S_I(p-q)^{NP} D^A_{\mu\nu}(q) \gamma^\nu\,. \quad (6.1.24)$$

$$-i\Sigma^Z_{IJ}(\not{p})^{NP}_{\text{weak}} = \delta_{IJ} \frac{g_W^2 M_Z^2}{16 M_W^2} \gamma^\mu (\lambda_I + \gamma_5)$$
$$\times \int \frac{\mathrm{d}^4 q}{(2\pi)^4 i}\, S_I(p-q)^{NP} D^Z_{\mu\nu}(q) \gamma^\nu (\lambda_I + \gamma_5)\,, \quad (6.1.25)$$

$$-i\Sigma^{\chi_3}_{IJ}(\not{p})^{NP}_{\text{weak}} = -\delta_{IJ} \frac{g_W^2 m_I^2}{4 M_W^2} \gamma_5 \int \frac{\mathrm{d}^4 q}{(2\pi)^4 i}\, S_I(p-q)^{NP} D^{\chi_3}(q) \gamma_5\,, \quad (6.1.26)$$

$$-i\Sigma^{\phi}_{IJ}(\not p)^{NP}_{\rm weak} = \delta_{IJ}\frac{g_W^2 m_I^2}{4M_W^2}\int\frac{{\rm d}^4q}{(2\pi)^4 i}\,S_I(p-q)^{NP}D^{\phi}(q)\,, \tag{6.1.27}$$

$$\begin{aligned}-i\Sigma^{W^\pm}_{IJ}(\not p)^{NP}_{\rm weak} = \frac{g_W^2}{8}\sum_{i=1}^{N}V_{Ii}(V^\dagger)_{iJ}\gamma^\mu(1-\gamma_5)\\ \times\int\frac{{\rm d}^4q}{(2\pi)^4 i}\,S_i(p-q)^{NP}D^W_{\mu\nu}(q)\gamma^\nu(1-\gamma_5)\,,\end{aligned} \tag{6.1.28}$$

$$\begin{aligned}-i\Sigma^{\chi^\pm}_{IJ}(\not p)^{NP}_{\rm weak} = -\frac{g_W^2}{8M_W^2}\sum_{i=1}^{N}V_{Ii}(V^\dagger)_{iJ}\left[(m_i-m_I)+(m_i+m_I)\gamma_5\right]\\ \times\int\frac{{\rm d}^4q}{(2\pi)^4 i}\,S_i(p-q)^{NP}D^{\chi}(q)\left[(m_J-m_i)+(m_J+m_i)\gamma_5\right],\end{aligned} \tag{6.1.29}$$

where $\lambda_I = -1 + 8(M_Z^2 - M_W^2)/(3M_Z^2)$. Once again the distinction between μ and m is maintained and the complications of Ward identity corrections are avoided by working in Landau gauge. Substituting the identity (C.1) and the propagators (Appendix B) restricted to Landau gauge into the self-energies (6.1.24)–(6.1.29), one obtains

$$-i\Sigma^{A}_{IJ}(\not p)^{NP}_{\rm weak} = \delta_{IJ}\frac{e^2}{9}\gamma^\mu i\langle\overline{\psi}_I\psi_I\rangle\sum_{k=0}^{\infty}C_k\mu_I^k\left[-\gamma\cdot\frac{\partial}{\partial p}\right]^k\left[\frac{g_{\mu\nu}}{p^2}-\frac{p_\mu p_\nu}{(p^2)^2}\right]\gamma^\nu, \tag{6.1.30}$$

$$\begin{aligned}-i\Sigma^{Z}_{IJ}(\not p)^{NP}_{\rm weak} = \delta_{IJ}\frac{g_W^2 M_Z^2}{16M_W^2}\gamma^\mu(\lambda_I+\gamma_5)i\langle\overline{\psi}_I\psi_I\rangle\sum_{k=0}^{\infty}C_k\mu_I^k\left[-\gamma\cdot\frac{\partial}{\partial p}\right]^k\\ \times\left[\frac{g_{\mu\nu}}{p^2-M_Z^2}-\frac{p_\mu p_\nu/M_Z^2}{p^2-M_Z^2}+\frac{p_\mu p_\nu/M_Z^2}{p^2}\right]\gamma^\nu(\lambda_I+\gamma_5)\,,\end{aligned} \tag{6.1.31}$$

$$-i\Sigma^{\chi_3}_{IJ}(\not p)^{NP}_{\rm weak} = -\delta_{IJ}\frac{g_W^2 m_I^2}{4M_W^2}\gamma_5 i\langle\overline{\psi}_I\psi_I\rangle\sum_{k=0}^{\infty}C_k\mu_I^k\left[-\gamma\cdot\frac{\partial}{\partial p}\right]^k\frac{1}{p^2}\gamma_5\,, \tag{6.1.32}$$

$$-i\Sigma^{\phi}_{IJ}(\not p)^{NP}_{\rm weak} = \delta_{IJ}\frac{g_W^2 m_I^2}{4M_W^2}i\langle\overline{\psi}_I\psi_I\rangle\sum_{k=0}^{\infty}C_k\mu_I^k\left[-\gamma\cdot\frac{\partial}{\partial p}\right]^k\frac{1}{m_\phi^2-p^2}\,. \tag{6.1.33}$$

$$-i\Sigma_{IJ}^{W^\pm}(\not{p})_{\text{weak}}^{NP} = \frac{g_W^2}{8}\sum_{i=1}^{N} V_{Ii}(V^\dagger)_{iJ}\,\gamma^\mu(1-\gamma_5)\,i\langle\overline{\psi}_i\psi_i\rangle\sum_{k=0}^{\infty} C_k\mu_i^k\left[-\gamma\cdot\frac{\partial}{\partial p}\right]^k$$
$$\times\left[\frac{g_{\mu\nu}}{p^2-M_W^2}-\frac{p_\mu p_\nu/M_W^2}{p^2-M_W^2}+\frac{p_\mu p_\nu/M_W^2}{p^2}\right]\gamma^\nu(1-\gamma_5)\,, \quad (6.1.34)$$

$$-i\Sigma_{IJ}^{\chi^\pm}(\not{p})_{\text{weak}}^{NP} = -\frac{g_W^2}{8M_W^2}\sum_{i=1}^{N} V_{Ii}(V^\dagger)_{iJ}\left[(m_i-m_I)+(m_i+m_I)\gamma_5\right]$$
$$\times i\langle\overline{\psi}_i\psi_i\rangle\sum_{k=0}^{\infty} C_k\mu_i^k\left[-\gamma\cdot\frac{\partial}{\partial p}\right]^k\frac{1}{p^2}\left[(m_J-m_i)+(m_J+m_i)\gamma_5\right]. \quad (6.1.35)$$

Using the series identities (C.3)–(C.9) one then obtains equations (6.1.36)–(6.1.41). Once again use has been made of the $O(1/M^2)$ approximations for the series (C.7)–(C.9). The t-quark is ignored so $I,\, J = u,\, c$.

$$\Sigma_{IJ}^{A}(\not{p})_{\text{weak}}^{NP} = -\delta_{IJ}\frac{4e^2\langle\overline{\psi}_I\psi_I\rangle}{9p^2}\mathbf{1}\,, \quad (6.1.36)$$

$$\Sigma_{IJ}^{Z}(\not{p})_{\text{weak}}^{NP} = \delta_{IJ}\frac{g_W^2\langle\overline{\psi}_I\psi_I\rangle}{384M_W^2}\left\{6(\lambda_I^2-1)\mathbf{1}-(\lambda_I^2+1)\frac{\mu_I(\mu_I^2-4p^2)}{(p^2)^2}\not{p}\right.$$
$$\left.+2\lambda_I\frac{\mu_I(\mu_I^2-4p^2)}{(p^2)^2}\gamma_5\not{p}\right\}, \quad (6.1.37)$$

$$\Sigma_{IJ}^{\chi_3}(\not{p})_{\text{weak}}^{NP} = -\delta_{IJ}\frac{g_W^2 m_I^2\langle\overline{\psi}_I\psi_I\rangle}{48M_W^2p^2}\left[1-\frac{\mu_I}{2p^2}\not{p}\right], \quad (6.1.38)$$

$$\Sigma_{IJ}^{\phi}(\not{p})_{\text{weak}}^{NP} = 0\,, \quad (6.1.39)$$

$$\Sigma_{IJ}^{W^\pm}(\not{p})_{\text{weak}}^{NP} = -\frac{g_W^2}{96M_W^2(p^2)^2}\sum_{i=1}^{N} V_{Ii}(V^\dagger)_{iJ}\,\langle\overline{\psi}_i\psi_i\rangle\mu_i(\mu_i^2-4p^2)(\not{p}+\gamma_5\not{p})\,, \quad (6.1.40)$$

$$\Sigma^{\chi^\pm}_{IJ}(\not p)^{NP}_{\text{weak}} = -\frac{g_W^2}{96M_W^2p^2}\Bigg\{2(m_I+m_J)\sum_{i=1}^{N} V_{Ii}(V^\dagger)_{iJ}\, m_i\langle\overline{\psi}_i\psi_i\rangle \mathbf{1}$$

$$-\frac{1}{p^2}\sum_{i=1}^{N} V_{Ii}(V^\dagger)_{iJ}\, \mu_i(m_i^2+m_Im_J)\langle\overline{\psi}_i\psi_i\rangle\not p$$

$$-2(m_I-m_J)\sum_{i=1}^{N} V_{Ii}(V^\dagger)_{iJ}\, m_i\langle\overline{\psi}_i\psi_i\rangle\gamma_5$$

$$-\frac{1}{p^2}\sum_{i=1}^{N} V_{Ii}(V^\dagger)_{iJ}\, \mu_i(m_i^2-m_Im_J)\langle\overline{\psi}_i\psi_i\rangle\gamma_5\not p\Bigg\}, \quad (6.1.41)$$

From equations (6.1.36)-(6.1.41) one collects together the $\mathbf{1}$, $\not p$, γ_5 and $\gamma_5\not p$ components of the self-energy

$$\Sigma_{\mathbf{1},IJ}(p^2)^{NP}_{\text{weak}} = -\delta_{IJ}\frac{e^2\langle\overline{\psi}_I\psi_I\rangle}{9p^2} + \delta_{IJ}\frac{g_W^2\langle\overline{\psi}_I\psi_I\rangle}{192M_W^2p^2}\left[3(\lambda_I^2-1)p^2-4m_I^2\right]$$

$$-\frac{g_W^2(m_I+m_J)}{48M_W^2p^2}\sum_{i=1}^{N} V_{Ii}(V^\dagger)_{iJ}\, m_i\langle\overline{\psi}_i\psi_i\rangle, \quad (6.1.42)$$

$$\Sigma_{\gamma,IJ}(p^2)^{NP}_{\text{weak}} = -\delta_{IJ}g_W^2(\lambda_I^2+1)\frac{\mu_I(\mu_I^2-4p^2)\langle\overline{\psi}_I\psi_I\rangle}{384M_W^2p^4} + \delta_{IJ}\frac{g_W^2m_I^2\mu_I\langle\overline{\psi}_I\psi_I\rangle}{96M_W^2p^4}$$

$$+\frac{g_W^2}{96M_W^2p^4}\sum_{i=1}^{N} V_{Ii}(V^\dagger)_{iJ}\, \mu_i(m_i^2+m_Im_J-\mu_i^2+4p^2)\langle\overline{\psi}_i\psi_i\rangle, \quad (6.1.43)$$

$$\Sigma_{5,IJ}(p^2)^{NP}_{\text{weak}} = -\frac{g_W^2(m_I-m_J)}{48M_W^2p^2}\sum_{i=1}^{N} V_{Ii}(V^\dagger)_{iJ}\, m_i\langle\overline{\psi}_i\psi_i\rangle, \quad (6.1.44)$$

$$\Sigma_{5\gamma,IJ}(p^2)^{NP}_{\text{weak}} = \delta_{IJ}\frac{g_W^2\lambda_I\mu_I(\mu_I^2-4p^2)\langle\overline{\psi}_I\psi_I\rangle}{192M_W^2p^4}$$

$$+\frac{g_W^2}{96M_W^2p^4}\sum_{i=1}^{N} V_{Ii}(V^\dagger)_{iJ}\, \mu_i(m_i^2-m_Im_J-\mu_i^2+4p^2)\langle\overline{\psi}_i\psi_i\rangle. \quad (6.1.45)$$

In the following section, the $\mathbf{1}$ and $\not p$ components (6.1.42) and (6.1.43) are inserted into the equation for the mass difference to obtain an equation for the constituent mass of charge +2/3 quarks.

Quark Flavour	Current Mass (GeV)	Constituent Mass (GeV)	Condensate $\langle\bar{q}q\rangle$ (GeV^3)
u	0.006	0.326	0.016
d	0.010	0.328	0.016
s	0.200	0.510	0.0128
c	1.350	1.726	0.0018
b	4.730	4.730	0.00066

Table 6.1. Accepted values for the current and constituent masses, and the quark condensates are listed for the first five flavours of quarks.[41]

6.2 The Contribution of the Nonperturbative Electroweak Self–Energies to the Mass Difference

Substitution of the **1** and $\not{p}$ components (6.1.20) and (6.1.21) of the electroweak self–energy are inserted into (4.2.10) along with the nonperturbative component of QCD represented by $m^3_{\mathrm{dyn},n}$ as in (5.2.16) to obtain the equation for the constituent mass of the charge $-1/3$ quarks.

$$\mu_i = m_i + \frac{m^3_{\mathrm{dyn},i}}{\mu_i} + \frac{e^2\langle\bar{\psi}_i\psi_i\rangle}{36\mu_i^2} - \frac{g_W^2\langle\bar{\psi}_i\psi_i\rangle}{384M_W^2\mu_i}\left[3(3\lambda_i^2-1)\mu_i - 4m_i^2\right]$$
$$+\frac{g_W^2}{96M_W^2\mu_i^3}\sum_{I=1}^{N}(V^\dagger)_{iI}V_{Ii}\left[4m_I m_i\mu_i - \mu_I(m_I^2-\mu_I^2+m_i^2+4\mu_i^2)\right]\langle\bar{\psi}_I\psi_I\rangle \quad (6.2.1)$$

If lowercase letters become uppercase letters in (4.2.10) then one obtains the equation for the mass difference for the charge $+2/3$ quarks. The **1** and $\not{p}$ components (6.1.42) and (6.1.43) are inserted into this equation along with the QCD contribution to obtain the equation for the constituent mass of the charge $+2/3$ quarks

$$\mu_I = m_I + \frac{m^3_{\mathrm{dyn},I}}{\mu_I^2} + \frac{e^2\langle\bar{\psi}_I\psi_I\rangle}{9\mu_I^2} - \frac{g_W^2\langle\bar{\psi}_I\psi_I\rangle}{384M_W^2\mu_I^2}\left[3(3\lambda_I^2-1)\mu_I^2 - 4m_I^2\right]$$
$$+\frac{g_W^2}{96M_W^2\mu_I^3}\sum_{I=1}^{N}V_{Ii}(V^\dagger)_{iI}\left[4m_i m_I\mu_I - \mu_i(m_i^2-\mu_i^2+m_I^2+4\mu_I^2)\right]\langle\bar{\psi}_i\psi_i\rangle \quad (6.2.2)$$

Quark	Electro- magnetic (e^2)	Weak (g_W^2)
u	1.51×10^{-3}	-1.55×10^{-8}
d	3.77×10^{-4}	-2.92×10^{-8}
s	1.19×10^{-4}	9.03×10^{-9}
c	8.64×10^{-7}	-4.46×10^{-9}
b	3.50×10^{-8}	8.08×10^{-11}

Table 6.2. The contributions in GeV of the electromagnetic and weak interactions in equation (6.2.1) for $i = d, s, b$ and (6.2.2) for $I = u, c$.

Values in the accepted range for the current and constituent masses and the quark condensates are listed in Table 6.1.[41] By substituting these values into the right hand sides of equations (6.2.1) and (6.2.2), one obtains the contribution of the electromagnetic and weak interactions to the constituent mass as listed in Table 6.2.

From Table 6.2 one notes that, as expected, the electroweak contributions are quite negligible. For example, the magnitude of the electromagnetic contribution to μ_d is less that 0.4 MeV, and the weak contribution is four orders of magnitude smaller.

Such estimates, as noted above, assume that standard-model fermion condensates arise *only* via the nonperturbative content of the QCD vacuum. Consequently, light-quark condensates are $O(\Lambda_{QCD}^3)$ quantities, as determined from phenomenological applications of QCD sum-rules to low energy hadronic physics,[5] while the t-quark condensate obtained from (6.1.19) is negligible.

CHAPTER SEVEN
THE $\langle\bar{t}t\rangle$ CONTRIBUTION TO THE d–QUARK MASS

7.1 The Restriction on a Large t–Quark Condensate

Recent models have proposed the t-quark condensate as a substitute for the Higgs coupling as a mechanism for generating masses.[12,13,43] In these models the t-quark condensate is produced by interactions that are beyond the standard model. These models suggest that $\langle\bar{t}t\rangle$ could be as large as Λ^3 where $\Lambda \simeq 10^{15} - 10^{19}$ GeV is a Nambu–Jona-Lasinio cutoff parameter.[13]

In this chapter the t-quark condensate is assumed to be very large. The calculation is done entirely within the context of the Standard Model but the constraint on the value of $\langle\bar{t}t\rangle$ imposed by the heavy quark expansion (6.1.19) is no longer imposed. The question is simply: What happens if $\langle\bar{t}t\rangle$ is allowed to be very large?

The t-quark condensate contributes to the charge $-1/3$ quarks only through $W^{\pm}$ or $\chi^{\pm}$ exchange (Figure 7.1 and 7.2). The unitary gauge, in which the contribution from the diagram in Figure 7.2 vanishes, is used. For simplicity, consider the d-quark. The mass relation for the d-quark is

$$\mu_d = m_d + m_{\rm dyn}^{\rm QCD} - \left[\Sigma_{1,dd}(\mu_d^2)_{\rm weak}^{NP} + \mu_d \Sigma_{\gamma,dd}(\mu_d^2)_{\rm weak}^{NP}\right]_{\langle\bar{t}t\rangle}, \qquad (7.1.1)$$

where only the $\langle\bar{t}t\rangle$ contribution to the electroweak self-energy is considered. It is assumed that all electroweak contributions except the t quark contribution are

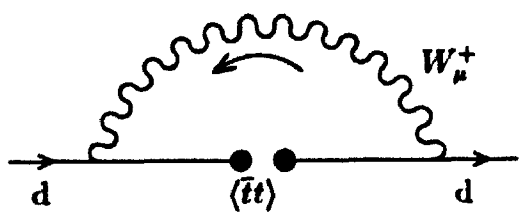


Figure 7.1 The condensate $\langle\bar{t}t\rangle$ contributes to the d–quark self–energy by exchange of a W_μ^+ gauge boson.

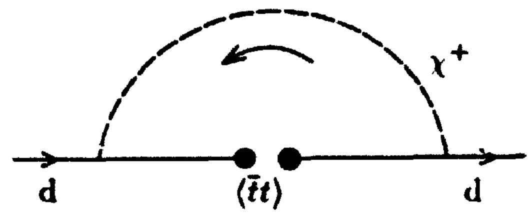


Figure 7.2 The condensate $\langle\bar{t}t\rangle$ contributes to the d–quark self–energy by exchange of a χ_μ^+. This contribution vanishes in the unitary gauge.

negligible. In Chapter 6 it was shown that these contributions are, indeed, negligible. Moreover, the t–quark condensate cannot contribute to the mass of the charge +2/3 quarks. If all other electroweak contributions are negligible, then

$$\mu_u = m_u + m_{\mathrm{dyn}}^{\mathrm{QCD}} . \tag{7.1.2}$$

The QCD contribution is assumed to be the same for both the u–and d–quark. Taking the difference between (7.1.1) and (7.1.2), one obtains

$$(m_d - m_u) - (\mu_d - \mu_u) = \left[\Sigma_{1,dd}(\mu_d^2)_{\mathrm{weak}}^{NP} + \mu_d \Sigma_{\gamma,dd}(\mu_d^2)_{\mathrm{weak}}^{NP}\right]_{\langle\bar{t}t\rangle} . \tag{7.1.3}$$

Estimated values for the current and constituent masses of the u–and d–quark are given in Table 6.1. With these values, the difference between the current masses is $(m_d - m_u) \approx 4$ MeV, and the difference between the constituent masses is $(\mu_d - \mu_u) \approx$ 2 MeV. This means that the right hand side of (7.1.3) must be of order 1 MeV. In order to obtain an order of magnitude calculation, take the absolute value of both sides of (7.1.3)

$$1 - 2\ \mathrm{MeV} > \left|\Sigma_{1,dd}(\mu_d^2)_{\mathrm{weak}}^{NP} + \mu_d \Sigma_{\gamma,dd}(\mu_d^2)_{\mathrm{weak}}^{NP}\right|_{\langle\bar{t}t\rangle} . \tag{7.1.4}$$

Thus, equation (7.1.4) imposes a rough limit on the value of the $\langle\bar{t}t\rangle$ condensate. In order to proceed further, the contribution of $\langle\bar{t}t\rangle$ to the d-quark self-energy must be computed.

7.2 The $\langle\bar{\mathbf{t}}\mathbf{t}\rangle$ Contribution to the d–Quark Self–Energy

The $\langle\bar{t}t\rangle$ contribution to the d-quark self-energy is calculated in the same manner as outlined in Sections 5.1 and 5.2. The d-quark self-energy contains the ground state expectation value of the normal ordered product of the t-quark field

$$\Sigma^{W}_{dd}(\not{p})^{NP}_{\text{weak}} = \frac{g^2_W}{8}(V^{-1})_{dt}V_{td}\gamma^{\mu}(1-\gamma_5)\int \mathrm{d}^4x\, e^{ip\cdot x}\langle\tilde{0}|:t(x)\bar{t}(0):|\tilde{0}\rangle$$

$$\times\langle 0|T\left[W_{\mu}(x)W_{\nu}(0)\right]|0\rangle\gamma^{\nu}(1-\gamma_5)\,. \quad (7.2.1)$$

In the unitary gauge, the $W^{\pm}$ propagator has the form

$$\langle 0|T\left[W_{\mu}(x)W_{\nu}(0)\right]|0\rangle = \int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{e^{-ik\cdot x}}{k^2-M^2_W+i\epsilon}\left(g_{\mu\nu}-\frac{k_{\mu}k_{\nu}}{M^2_W}\right)\,. \quad (7.2.2)$$

The nonperturbative propagator for the t quark $\langle\tilde{0}|:t(x)\bar{t}(0):|\tilde{0}\rangle$ cannot be expanded as a series (5.2.7) because the series expansion is invalid at the mass scale of the t-quark. An alternative form must be found for the ground state expectation value of the normal ordered product of the t quark fields.

First note that the field $t(x)$ must be a solution of the equation

$$(i\not{D}-m_t)t(x) = 0\,. \quad (7.2.3)$$

Where $\not{D} = \not{\partial} + \not{A}(x)$ is the covariant derivative. Multiplying on the right by $t(0)$ and then normal ordering, one can rewrite (7.2.3) as

$$(i\not{\partial} - \mu_t) :t(x)\bar{t}(0): +:t(x)\not{A}(x)\bar{t}(0):= 0\,, \tag{7.2.4}$$

since normal ordering, unlike time ordering, does not introduce any additional coordinate-space dependence. Take the ground state expectation value of (7.2.4):

$$(i\not{\partial} - \mu_t)\langle\tilde{0}|:t(x)\bar{t}(0):|\tilde{0}\rangle + \langle\tilde{0}|:t(x)\not{A}(x)\bar{t}(0):|\tilde{0}\rangle = 0\,. \tag{7.2.5}$$

The first term $\langle\tilde{0}|:t(x)\bar{t}(0):|\tilde{0}\rangle$ contains the dimension-3 condensate $\langle\bar{t}t\rangle$ plus higher dimensional condensates.[9] The second term $\langle\tilde{0}| : t(x)\not{A}(x)\bar{t}(0) : |\tilde{0}\rangle$ involves only condensates of dimension 5 or greater.[5,9] Condensates of different dimensions are linearly independent. Because the only term involving a dimension-3 condensate in (7.2.5) is the first term, it must vanish identically

$$(i\not{\partial} - \mu_t)\langle\tilde{0}|:t(x)\bar{t}(0):|\tilde{0}\rangle = 0\,. \tag{7.2.6}$$

where it is understood that only the dimension-3 term of $\langle\tilde{0}|:t(x)\bar{t}(0):|\tilde{0}\rangle$ will be considered. Thus, $\langle\tilde{0}|:t(x)\bar{t}(0):|\tilde{0}\rangle$ must be a solution of the free Dirac equation for a fermion of mass μ_t. The condition

$$\lim_{x\to 0}\langle\tilde{0}|:t_i^\alpha(x)\bar{t}_j^\beta(0):|\tilde{0}\rangle = -\frac{1}{12}\delta_{ij}\delta_{\alpha\beta}\langle\bar{t}t\rangle. \tag{7.2.7}$$

where i and α are Dirac and colour indices, respectively, is the initial condition for the differential equation (7.2.6). This is consistent with the definition

$$\lim_{x\to 0}\langle\tilde{0}|:t_i^\alpha(x)\bar{t}_i^\alpha(0):|\tilde{0}\rangle = -\langle\bar{t}t\rangle\,. \tag{7.2.8}$$

The solution to (7.2.6) that satisfies the initial condition (7.2.7) is[11]

$$\langle\tilde{0}|:t(x)\bar{t}(0):|\tilde{0}\rangle = -\frac{\langle\bar{t}t\rangle}{6\mu_t^2}(i\partial\!\!\!/ + \mu_t)\left[\frac{J_1(\mu_t\sqrt{x^2})}{\sqrt{x^2}}\right], \tag{7.2.9}$$

which corresponds *identically* to the series solution (5.2.7) obtained by OPE methods. It is convenient to introduce a momentum space representation of the non-perturbative propagator (7.2.9). In Reference 17, it is shown that the appropriate representation is

$$\langle\tilde{0}|:t(x)\bar{t}(0):|\tilde{0}\rangle = \int d^4k\, e^{-ik\cdot x}(k\!\!\!/ + \mu_t)\mathcal{F}_t(k). \tag{7.2.10}$$

Comparison of (7.2.9) and (7.2.10) shows that $\mathcal{F}_t(k)$ must be defined by the relationship

$$\int d^4k\, e^{-ik\cdot x}\mathcal{F}_t(k) \equiv -\frac{\langle\bar{t}t\rangle J_1(\mu_t\sqrt{x^2})}{6\mu_t^2\sqrt{x^2}}. \tag{7.2.11}$$

Upon substitution of (7.2.2) and (7.2.10) into (7.2.1), one finds that the $\langle\bar{t}t\rangle$ contribution to the d-quark self energy is

$$\Sigma_{dd}^{W}(p\!\!\!/)_{\mathrm{weak}}^{NP} = \frac{g_W^2}{8}(V^\dagger)_{dt}V_{td}\gamma^\mu(1-\gamma_5)\int d^4k\,\frac{(k\!\!\!/ + \mu_t)\mathcal{F}_t(k)}{(p-k)^2 - M_W^2 + i\epsilon}$$
$$\times\left[g_{\mu\nu} - \frac{(p-k)_\mu(p-k)_\nu}{M_W^2}\right]\gamma^\nu(1-\gamma_5). \tag{7.2.12}$$

In the evaluation of (7.2.12), it is useful to note that because (7.2.10) is a solution of the free Dirac equation[11]

$$(k^2 - \mu_t^2)\mathcal{F}_t(k) = 0, \tag{7.2.13}$$

thereby allowing the replacement of each factor of k^2 with a factor of μ_t^2 in the integrand of (7.2.12).[17] After some algebra and use of (7.2.13), one obtains

$$\Sigma^W_{dd}(\not{p})^{NP}_{\text{weak}} = -\frac{g_W^2}{4M_W^2}(V^\dagger)_{dt}V_{td}(1+\gamma_5)$$
$$\times\left\{(2M_W^2+\mu_t^2-p^2)\int \mathrm{d}^4k\,\frac{\mathcal{F}_t(k)\not{k}}{(p-k)^2-M_W^2+i\epsilon}\right.$$
$$\left.+2\not{p}\int \mathrm{d}^4k\,\frac{\mathcal{F}_t(k)p\cdot k}{(p-k)^2-M_W^2+i\epsilon}-2\mu_t^2\not{p}\int \mathrm{d}^4k\,\frac{\mathcal{F}_t(k)}{(p-k)^2-M_W^2+i\epsilon}\right\}. \quad (7.2.14)$$

Using the identity (D.22)

$$\int \mathrm{d}^4k\,\frac{\mathcal{F}_t(k)\not{k}}{(p-k)^2-M_W^2+i\epsilon}=\frac{\not{p}}{p^2}\int \mathrm{d}^4k\,\frac{\mathcal{F}_t(k)p\cdot k}{(p-k)^2-M_W^2+i\epsilon}, \quad (7.2.15)$$

one obtains

$$\Sigma^W_{dd}(\not{p})^{NP}_{\text{weak}} = -\frac{g_W^2}{4M_W^2}(1+\gamma_5)\not{p}(V^\dagger)_{dt}V_{td}\left\{\frac{(2M_W^2+\mu_t^2+p^2)}{p^2}\right.$$
$$\left.\times\int \mathrm{d}^4k\,\frac{\mathcal{F}_t(k)p\cdot k}{(p-k)^2-M_W^2+i\epsilon}-2\mu_t^2\int \mathrm{d}^4k\,\frac{\mathcal{F}_t(k)}{(p-k)^2-M_W^2+i\epsilon}\right\}. \quad (7.2.16)$$

The integrals appearing in (7.2.16) are solved in Appendix D. Using (D.12) and (D.19), one obtains

$$\Sigma^W_{dd}(\not{p})^{NP}_{\text{weak}} = -\frac{g_W^2}{4M_W^2}(V^\dagger)_{dt}V_{td}(1+\gamma_5)\not{p}$$
$$\times\left\{\frac{(2M_W^2+\mu_t^2+p^2)}{p^2}\left[\frac{\langle\bar{t}t\rangle}{24\mu_t}+\frac{1}{2}(p^2+\mu_t^2-M_W^2)I(p^2)\right]-2\mu_t^2I(p^2)\right\}, \quad (7.2.17)$$

where $I(p^2)$ is defined in equation (D.12) to be

$$I(p^2)=-\frac{\langle\bar{t}t\rangle}{24\mu_t^3p^2}\left[(p^2+\mu_t^2-M_W^2)-\sqrt{(p^2-\mu_t^2-M_W^2)^2-4\mu_t^2M_W^2}\right]. \quad (7.2.18)$$

The domain of the function $I(p^2)$ is given in (D.14). The $\langle\bar{t}t\rangle$ condensate contributes to the $\not{p}$ and $\gamma_5\not{p}$ but not the **1** and γ_5 components of the d-quark self-energy. Therefore, by equation (7.1.4) the restriction on $\langle\bar{t}t\rangle$ is imposed only on the $\not{p}$ component

$$\Sigma^{W}_{\gamma,dd}(p^2)^{NP}_{\mathrm{weak}} = -\frac{g_W^2}{4M_W^2}(V^\dagger)_{dt}V_{td}$$
$$\times\left\{\frac{(2M_W^2+\mu_t^2+p^2)}{p^2}\left[\frac{\langle\bar{t}t\rangle}{24\mu_t}+\frac{1}{2}(p^2+\mu_t^2-M_W^2)I(p^2)\right]-2\mu_t^2 I(p^2)\right\}, \quad (7.2.19)$$

If the values of the momentum are restricted to the domain $p^2 < (\mu_t - M_W)^2$, then (7.2.19) can be expanded about $p^2 = 0$. Upon setting $p^2 = \mu_d^2$, which is in the domain $\mu_d^2 < (\mu_t - M_W)^2$, one obtains

$$\Sigma^{W}_{\gamma,dd}(\mu_d^2)^{NP}_{\mathrm{weak}} = -\frac{g_W^2(V^\dagger)_{dt}V_{td}\,\mu_t\langle\bar{t}t\rangle}{32M_W^2(\mu_t^2-M_W^2)^2}$$
$$\times\left[(\mu_t^2-2M_W^2)-\frac{(9M_W^4-4\mu_t^2M_W^2+\mu_t^4)}{3(\mu_t^2-M_W^2)^2}\mu_d^2+O(\mu_d^4)\right]. \quad (7.2.20)$$

This is the t–quark condensate contribution to the d quark self-energy.

7.3 The Value of the t–Quark Condensate

Because the t–quark contributes to the $\not{p}$ but not the **1** component of the d–quark self energy, the relation (7.1.4) becomes

$$2\ \mathrm{MeV} > \left|\mu_d\Sigma^{W}_{\gamma,dd}(\mu_d^2)^{NP}_{\mathrm{weak}}\right|_{\langle\bar{t}t\rangle}. \quad (7.3.1)$$

Substitution of (7.2.20) into (7.3.1) yields the condition

$$2\ \mathrm{MeV} > \left|-\frac{g_W^2(V^\dagger)_{dt}V_{td}\,\mu_d\mu_t\langle\bar{t}t\rangle}{32M_W^2(\mu_t^2-M_W^2)^2}\right.$$
$$\left.\times\left[(\mu_t^2-2M_W^2)-\frac{(9M_W^4-4\mu_t^2M_W^2+\mu_t^4)}{3(\mu_t^2-M_W^2)^2}\mu_d^2+O(\mu_d^4)\right]\right|. \quad (7.3.2)$$

		V_{td}				
		0.003	0.007	0.011	0.015	0.019
	120	105.62	19.39	7.86	4.22	2.63
	130	65.53	12.04	4.87	2.62	1.63
μ_t	140	57.99	10.65	4.31	2.32	1.45
	150	56.44	10.37	4.20	2.26	1.41
	160	57.03	10.47	4.24	2.28	1.42

Table 7.1 The upper limits of $|\langle\bar{t}t\rangle| \times 10^9$ GeV^3 computed from equation (7.3.3) for various values of the CKM mixing parameter V_{td} and the t–quark mass μ_t (GeV).

Rearranging this equation, one obtains an explicit restriction on $|\langle\bar{t}t\rangle|$ in terms of the measurable parameters of electroweak theory.

$$|\langle\bar{t}t\rangle| < \left| \frac{0.064 M_W^2(\mu_t^2 - M_W^2)^2}{g_W^2 (V^\dagger)_{dt} V_{td}\, \mu_d \mu_t (\mu_t^2 - 2M_W^2)} \times \left[1 + \frac{(9M_W^4 - 4\mu_t^2 M_W^2 + \mu_t^4)}{3(\mu_t^2 - 2M_W^2)(\mu_t^2 - M_W^2)^2} \mu_d^2 + O(\mu_d^4) \right] \right| . \quad (7.3.3)$$

The results of equation (7.3.3) are summarized in Table 7.1. The range of values for V_{td} that appear in Table 7.1 represents the uncertainty in the measurement of the elements of the CKM matrix.[42] The largest possible upper limit for $|\langle\bar{t}t\rangle|$ is 105.62×10^9 GeV^3, which occurs for the minimum known value $V_{td} = 0.003$ and a t–quark mass of $\mu_t = 120$ GeV. The smallest possible upper limit for $|\langle\bar{t}t\rangle|$ is about 1.41×10^9 GeV^3, which occurs for the maximum known value $V_{td} = 0.019$ and a t–quark mass of $\mu_t = 150$ GeV. Within the scheme assumed in this thesis, both of these bounds, while quite large, would appear to exclude a t–quark condensate referenced to a Nambu–Jona-Lasinio cut–off at Planck-momentum scales $\langle\bar{t}t\rangle \sim \mu_t^2 \Lambda_{\text{Planck}}$.[13]

CHAPTER EIGHT
EXAMPLE OF AN OFF–DIAGONAL TWO–POINT FUNCTION

8.1 The Off-Diagonal Inverse Propagator

Electroweak CKM mixing generates the possibility of off–diagonal contributions to the fermion inverse propagator. Both the $W^{\pm}$ and its scalar partner $\chi^{\pm}$ mediate the transition of an s–quark to a d–quark. To simplify the discussion, consider the unitary gauge. In this gauge the propagator for $\chi^{\pm}$ vanishes and the $W^{\pm}$ is entirely responsible for the off–diagonal transition. The $W^{\pm}$–fermion–antifermion vertices (B.8) and (B.9) have the gamma matrix structure $(1+\gamma_5)\gamma_\mu$. Therefore, the ds self–energy mediated by the $W^{\pm}$ has the form

$$\Sigma_{ds}(p) = H(p^2)(1+\gamma_5)\not{p}\,. \tag{8.1.1}$$

The components of the self energy are

$$\begin{aligned} \Sigma_{\gamma,ds}(p^2) &= H(p^2)\,, \\ \Sigma_{5\gamma,ds}(p^2) &= H(p^2)\,. \end{aligned} \tag{8.1.2}$$

From (3.1.19)–(3.1.22) the off-diagonal components of the inverse propagator are

$$\begin{aligned} K_{1,ds}(p^2) &= A\,, \\ K_{\gamma,ds}(p^2) &= B + H(p^2)\,, \\ K_{5,ds}(p^2) &= C\,, \\ K_{5\gamma,ds}(p^2) &= D + H(p^2)\,. \end{aligned} \tag{8.1.3}$$

The constants A, B, C and D are the off-diagonal elements of the renormalization constants

$$
\begin{aligned}
A &= -\frac{1}{2}\left(\mathbf{Z}_L^{1/2\dagger}\mathbf{M}_0\mathbf{Z}_R^{1/2} + \mathbf{Z}_R^{1/2\dagger}\mathbf{M}_0\mathbf{Z}_L^{1/2}\right)_{ds}, \\
B &= \frac{1}{2}\left(\left|\mathbf{Z}_L^{1/2}\right|^2 + \left|\mathbf{Z}_R^{1/2}\right|^2\right)_{ds}, \\
C &= -\frac{1}{2}\left(\mathbf{Z}_L^{1/2\dagger}\mathbf{M}_0\mathbf{Z}_R^{1/2} - \mathbf{Z}_R^{1/2\dagger}\mathbf{M}_0\mathbf{Z}_L^{1/2}\right)_{ds}, \\
D &= \frac{1}{2}\left(\left|\mathbf{Z}_L^{1/2}\right|^2 - \left|\mathbf{Z}_R^{1/2}\right|^2\right)_{ds}.
\end{aligned}
\tag{8.1.4}
$$

For very small values of p^2, the function $H(p^2)$ can be expanded to first order in p^2

$$
H(p^2) = H(0) + p^2 H'(0). \tag{8.1.5}
$$

It is assumed that this truncated expansion is valid for $p^2 = \mu_d$ and $p^2 = \mu_s$. The components of the off-diagonal inverse propagator (8.1.3) become

$$
\begin{aligned}
K_{1,ds}(p^2) &= A, \\
K_{\gamma,ds}(p^2) &= B + H(0) + p^2 H'(0), \\
K_{5,ds}(p^2) &= C, \\
K_{5\gamma,ds}(p^2) &= D + H(0) + p^2 H'(0).
\end{aligned}
\tag{8.1.6}
$$

The constants A, B, C and D are determined by the off-diagonal renormalization conditions (3.2.17)–(3.2.20)[44]

$$
\begin{aligned}
K_{1,ds}(\mu_s^2) + \mu_s K_{\gamma,ds}(\mu_s^2) &= 0, \\
K_{5,ds}(\mu_s^2) + \mu_s K_{5\gamma,ds}(\mu_s^2) &= 0, \\
K_{1,ds}(\mu_d^2) + \mu_d K_{\gamma,ds}(\mu_d^2) &= 0, \\
K_{5,ds}(\mu_d^2) - \mu_d K_{5\gamma,ds}(\mu_d^2) &= 0.
\end{aligned}
\tag{8.1.7}
$$

Substitute (8.1.8) into (8.1.6) and solve for the constants:

$$\begin{aligned} A &= \quad \mu_s\mu_d(\mu_s+\mu_d)H'(0)\,, \\ B &= -H(0)-(\mu_s^2+\mu_s\mu_d+\mu_d^2)H'(0)\,, \\ C &= -\mu_s\mu_d(\mu_s-\mu_d)H'(0)\,, \\ D &= -H(0)-(\mu_s^2-\mu_s\mu_d+\mu_d^2)H'(0)\,. \end{aligned} \tag{8.1.8}$$

Substitution of (8.1.7) back into (8.1.5) yields the components of the off-diagonal inverse propagators:

$$\begin{aligned} K_{1,ds}(p^2) &= \quad \mu_s\mu_d(\mu_s+\mu_d)H'(0)\,, \\ K_{\gamma,ds}(p^2) &= -(\mu_s^2+\mu_s\mu_d+\mu_d^2)H'(0)+p^2H'(0)\,, \\ K_{5,ds}(p^2) &= -\mu_s\mu_d(\mu_s-\mu_d)H'(0)\,, \\ K_{5\gamma,ds}(p^2) &= -(\mu_s^2-\mu_s\mu_d+\mu_d^2)H'(0)+p^2H'(0)\,. \end{aligned} \tag{8.1.9}$$

The complete off-diagonal inverse propagator has the form

$$K_{ds}(\not p) = K_{1,ds}(p^2)\mathbf{1} + K_{\gamma,ds}(p^2)\not p + K_{5,ds}(p^2)\gamma_5 + K_{5\gamma,ds}(p^2)\gamma_5\not p\,. \tag{8.1.10}$$

Substitution of (8.1.9) into (8.1.10) yields the final form for the off-diagonal inverse propagator[14]

$$\begin{aligned} K_{ds}(\not p) = H'(0)\Big\{ &\left[p^2-\mu_s^2-\mu_d^2\right]\not p(1-\gamma_5) \\ &+\mu_s\mu_d\left[-\not p(1+\gamma_5)+\mu_s(1-\gamma_5)+\mu_d(1+\gamma_5)\right]\Big\}\,. \end{aligned} \tag{8.1.11}$$

The strength of the off-diagonal transition amplitude depends on the magnitude of $H'(0)$. The purely perturbative contribution arising from a t quark intermediate state (Figure 8.1) is given by[14]

$$H'(0)^P = \frac{g_W^2}{32\pi^2M_W^2}(V^\dagger)_{dt}V_{ts}\left(B_t+\frac{5}{12}\right)\,, \tag{8.1.12}$$

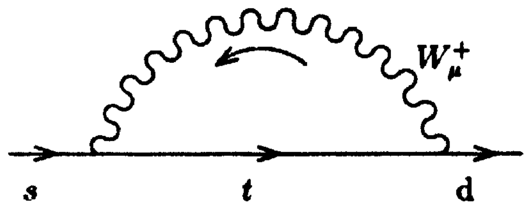


Figure 8.1 The perturbative contribution of an intermediate t-quark to the off-diagonal sd-element of the inverse propagator.

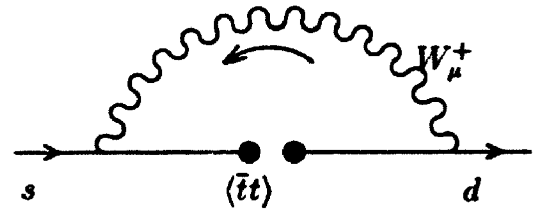


Figure 8.2 The $\langle \bar{t}t \rangle$ contribution to the off-diagonal sd-element of the inverse propagator.

with B_t between -1.0 and -1.3 for μ_t between 120 and 160 GeV.

If electroweak symmetry breaking is caused by a $\langle \bar{t}t \rangle$ condensate, then CKM mixing will permit the coupling of such a condensate not only to diagonal dd and ss two-point functions, as considered in the previous chapter, but also to sd off-diagonal two-point functions. In unitary gauge, the calculation of this contribution (Figure 8.2) is identical to the diagonal calculation that leads to equation (7.2.19), provided $(V^\dagger)_{dt} V_{td}$ in (7.2.19) is replaced with $(V^\dagger)_{dt} V_{ts}$.

$$\begin{aligned} H(p^2)^{NP} &= -\frac{g_W^2}{4M_W^2}(V^\dagger)_{dt}V_{ts} \\ &\times \left\{ \frac{(2M_W^2 + \mu_t^2 + p^2)}{p^2} \left[\frac{\langle \bar{t}t \rangle}{24\mu_t} + \frac{1}{2}(p^2 + \mu_t^2 - M_W^2) I(p^2) \right] - 2\mu_t^2 I(p^2) \right\}, \end{aligned} \quad (8.1.13)$$

Expanding this expression about $p^2 = 0$, as in (7.2.20), one finds that[45]

$$H'(0)^{NP} = \frac{g_W^2 (V^\dagger)_{dt} V_{ts} \langle \bar{t}t \rangle \mu_t^5}{96(\mu_t^2 - M_W^2)^4} \left[1 - 4\frac{M_W^2}{\mu_t^2} + 9\frac{M_W^4}{\mu_t^4} \right]. \quad (8.1.14)$$

8.2 The $|\Delta I| = 1/2$ Rule for Nonleptonic Decays of the Kaon

Evidence for the $|\Delta I| = 1/2$ rule in the nonleptonic decay of the kaon is found by studying the decays[46]

$$K_s^0 \to \pi^+\pi^- \qquad K_s^0 \to \pi^0\pi^0 \qquad K^+ \to \pi^+\pi^0 \,. \tag{8.2.1}$$

In these decays the two pion final state has zero angular momentum and is spatially symmetric. Because the pions have isospin $I(\pi) = 1$, the two pion state can have isospin $I(\pi\pi) = 0, 1, 2$. However, the $I(\pi\pi) = 1$ state is forbidden because it is antisymmetric under an interchange of the pions. Thus, the two pion states in the $K_s^0 \to \pi^+\pi^-$ and $K_s^0 \to \pi^0\pi^0$ decays can have either $I(\pi\pi) = 0$ or $I(\pi\pi) = 2$ and the two pion state in the $K^+ \to \pi^+\pi^0$ decay can have *only* $I(\pi^+\pi^0) = 2$. The isospin $I(\pi^+\pi^0) = 0$ is forbidden in the latter decay because $I_3(\pi^+\pi^0) = 1$.

The kaon isospin is $I(K) = 1/2$. If $I(\pi\pi) = 0$ then $|\Delta I| = 1/2$, and if $I(\pi\pi) = 2$ then $|\Delta I| = 3/2,\ 5/2$. If only $|\Delta I| = 1/2$ decays were allowed, then only a final two pion state with $I(\pi\pi) = 0$ would be allowed. In this case the branching ratio for K_s^0 would be

$$B(K_s^0) = \frac{\Gamma(K_s^0 \to \pi^+\pi^-)}{\Gamma(K_s^0 \to \pi^0\pi^0)} = 2\,, \tag{8.2.2}$$

and the $K^+ \to \pi^+\pi^0$ decay would be completely forbidden. The branching ratio is measured experimentally to be $B(K_s^0) = 2.186$, and the decay $K^+ \to \pi^+\pi^0$ does occur but is very strongly suppressed. Therefore, although $|\Delta I| = 1/2$ is not an exact rule, it is definitely enhanced over the $|\Delta I| = 3/2$ and $|\Delta I| = 5/2$ decays.

A kaon is composed of an s–quark, which has isospin $I(s) = 0$, and either a u–quark or a d–quark, both of which have $I(u, d) = 1/2$. If the s–quark undergoes an off-diagonal transition and becomes a d–quark then the isospin will change by $|\Delta I| = 1/2$, which is precisely the decay rule for kaon decays. Therefore, the off-diagonal inverse propagator $K_{ds}(\not{p})$ contributes *only* to the $|\Delta I| = 1/2$ decay of the kaon.

If $a_{1/2}$ is the amplitude for the $|\Delta I| = 1/2$ decay of a kaon, then[14,47]

$$a_{1/2} = \frac{3\sqrt{2}\, g_{Kqq}}{64\pi^2 f_\pi^2}(\mu_s - \mu_d) M^4 \left(1 - \frac{m_\pi^2}{m_K^2}\right) H'(0)\,. \tag{8.2.3}$$

where M is a phenomenological cut–off of order 2 GeV, $g_{Kqq} \approx 3.8$, and $f_\pi = 0.132$ GeV.

8.3 The $\langle \bar{t}t \rangle$ Contribution to the $|\Delta I| = 1/2$ Rule

The renormalized, perturbative off–diagonal self–energy (8.1.12) must contribute to $|\Delta I| = 1/2$ amplitudes.[47,48] However, it is well known that the subtractions required at two distinct mass–shells by the renormalization conditions (3.2.17)–(3.2.20) greatly diminish the magnitude of the perturbative contribution $H'(0)^P$ to strangeness–changing nonleptonic processes. Using the presently known range of CKM mixing angles[42] and values of μ_t between 120 and 160 GeV, one finds that $1 \times 10^{-11} < H'(0) < 2 \times 10^{-10}$ GeV^{-2}. In this case, equation (8.2.3) yields $3.7 \times 10^{-11} < a_{1/2} < 7.4 \times 10^{-10}$ GeV, which is three to four orders of magnitude smaller than the experimentally measured value $a_{1/2} \approx 3.5 \times 10^{-7}$ GeV. A more careful treatment[49] of the hadronization of $\Sigma_{d_s}^{ren}(p)$, based upon a direct comparison of the weak kaon axial-vector current $-i\int d^4x\, e^{iq\cdot x}\langle 0|T[A_\mu^3(x)H_W^{\Delta S=1}]|\bar{K}^0\rangle$ to the strong-interaction axial-vector current $i\sqrt{2}f_K q_\mu = -i\int d^4x\,\langle 0|A_\mu^{6+i7}(x)|\bar{K}^0\rangle$, leads to virtually the same conclusions: the perturbative contribution $H'(0)^P$ is between two and four orders of magnitude too small to account for the measured $K \to 2\pi$ decay rate.

The nonperturbative contribution $H'(0)^{NP}$ given in (8.1.14) can also be inserted into the $|\Delta I| = 1/2$ amplitude (8.2.3). If $\langle\bar{t}t\rangle$ is bounded from above by a magnitude sufficiently large to account for the amplitude $a_{1/2} = 3.5 \times 10^{-7}$ GeV, then the following constraint is obtained:

$$|\langle\bar{t}t\rangle| < \frac{2048\pi^2 f_\pi^2(\mu_t^2 - M_W^2)^4 M_W^2 a_{1/2}}{g_{Kqq}g_W^2\sqrt{2}(V^\dagger)_{dt}V_{ts}M^4\mu_t^5(\mu_s - \mu_d)} \times \frac{1}{\left(1 - \dfrac{m_\pi^2}{m_K^2}\right)\left(1 - 4\dfrac{M_W^2}{\mu_t^2} + 9\dfrac{M_W^4}{\mu_t^4}\right)}. \tag{8.3.1}$$

The results of equation (8.3.1) are summarized in Table 8.1. The largest upper limit for $|\langle\bar{t}t\rangle|$ is 36.55×10^8 GeV^3, which occurs for the minimum known values of the CKM parameters $V_{td} = 0.003$ and $V_{ts} = 0.029$ and a t-quark mass of $\mu_t = 160$ GeV. The smallest upper limit for $|\langle\bar{t}t\rangle|$ is 0.21×10^8 GeV, which occurs for the maximum known values of the CKM parameters $V_{td} = 0.019$ and $V_{ts} = 0.058$ and a t-quark mass of $\mu_t = 120$ GeV. These values of $|\langle\bar{t}t\rangle|$ are both within an order of magnitude of the upper limits established in Chapter 7. These results suggest that if the electroweak symmetry is broken by a $\langle\bar{t}t\rangle$ condensate, then a sufficiently large magnitude of the t quark condensate may be at least partly responsible for the enhancement of the $|\Delta I| = 1/2$ channels of nonleptonic interactions.

$\mu_t = 120$	V_{td} 0.003	0.007	0.011	0.015	0.019
V_{ts} 0.029	2.61	1.12	0.71	0.52	0.41
0.036	2.08	0.90	0.57	0.42	0.33
0.043	1.74	0.75	0.47	0.35	0.27
0.051	1.49	0.64	0.41	0.30	0.24
0.058	1.31	0.56	0.36	0.26	0.21

(a)

$\mu_t = 130$	V_{td} 0.003	0.007	0.011	0.015	0.019
V_{ts} 0.029	6.69	2.86	1.83	1.34	1.06
0.036	5.35	2.29	1.46	1.07	0.85
0.043	4.46	1.91	1.22	0.89	0.70
0.051	3.82	1.64	1.04	0.76	0.60
0.058	3.35	1.43	0.91	0.67	0.53

(b)

$\mu_t = 140$	V_{td} 0.003	0.007	0.011	0.015	0.019
V_{ts} 0.029	13.70	5.87	3.74	2.74	2.16
0.036	10.96	4.70	2.99	2.19	1.73
0.043	9.14	3.92	2.49	1.83	1.44
0.051	7.83	3.36	2.14	1.57	1.24
0.058	6.85	2.94	1.87	1.37	1.08

(c)

$\mu_t = 150$	V_{td} 0.003	0.007	0.011	0.015	0.019
V_{ts} 0.029	23.77	10.19	6.48	4.75	3.75
0.036	19.02	8.15	5.19	3.80	3.00
0.043	15.85	6.79	4.32	3.17	2.50
0.051	13.58	5.82	3.70	2.72	2.14
0.058	11.87	5.09	3.24	2.38	1.88

(d)

SUMMARY

The renormalization of electroweak theory can be performed consistently at a mass that differs from the Lagrangian mass. The renormalization point is interpreted as the constituent quark mass μ and the Lagrangian mass is interpreted as the current mass m. The renormalization yields an equation that determines the mass difference $(\mu - m)$ in terms of the nonperturbative component of the self-energy.

The use of the constituent mass necessitates the introduction of nonperturbative contributions to the quark self-energies. These contributions arise from the ground state expectation value of the normal-ordered product of fermionic fields. The dimension-3 quark condensate $\langle \overline{q}q \rangle$ dominates the expansion of the nonperturbative propagator $\langle \tilde{0} | : \psi(x)\overline{\psi}(0) : | \tilde{0} \rangle$. Therefore, the $\langle \overline{q}q \rangle$ contribution to the self-energy dominates the mass difference $(\mu - m)$. The $\langle \overline{q}q \rangle$ contribution of the strong interactions dominates the $\langle \overline{q}q \rangle$ contribution of the electroweak interactions. This calculation assumes that the t-quark condensate $\langle \bar{t}t \rangle$ is negligible due to the heavy quark expansion.

However, some recently proposed models that involve interactions beyond the standard model allow for a very large t-quark condensate. If $\langle \bar{t}t \rangle$ is very large then it will have an effect on processes that are strictly within the context of the standard model.

The magnitude of a large t quark condensate is limited by the mass difference between the u and d quarks. The $\langle \bar{t}t \rangle$ contribution to the self energy is at most of

CHAPTER NINE
CHIRAL GAUGE INTERACTIONS AND DYNAMICAL FERMION MASSES

9.1 Dynamical Origin of the Current Quark Mass?

Prior to the present chapter, only standard $SU(3) \times SU(2) \times U(1)$ gauge interactions have been considered. In this Chapter, some more speculative ideas regarding the possibility of dynamically generating masses in a larger grand unified theory are considered. The methodology of the previous Chapters should be applicable to any model in which symmetry breaking is driven by fermion-antifermion condensates.

In Chapter 8, it was shown that the condensate of the t-quark provides a contribution to the d-quark mass through the d-quark self energy involving exchange of a $W_\mu^\pm$ gauge boson. The fermion-antifermion-$W^\pm$ interaction is purely left handed and has the structure shown in equation (8.1.1). It was shown that the strong interaction dominates the contribution to the constituent mass of the d-quark. Within the context of a larger grand unified theory, however, a heavy fermion antifermion condensate may also provide a dynamical origin for the *current* quark masses. A gauge interaction that is beyond the standard model would be responsible for the appearance of such a condensate in the light quark self-energy.

The model considered below is assumed to have all of the relevant features required for a heavy fermion to contribute to the masses of light fermions. Assume that the model is chirally symmetric: the bare mass $m_i^{(0)}$, the mass counterterm

$\delta m_i^{(0)}$, and the renormalized mass m_i are all zero. Assume that the model contains a heavy fermion, which has a condensate $\langle \overline{f} f \rangle$ contributing to the mass of the lighter fermions through intergenerational mixing via both left-handed L_μ and right-handed R_μ massive gauge bosons beyond the standard model. Note that such chiral gauge interactions will not contribute to light fermion masses in a purely perturbative context; i.e., ν_e and ν_μ do not acquire masses from their $W^{\pm}$-mediated self-energy contributions involving transitions to e^- and μ^-, which are leptons of *nonzero* mass. In order for the discussion to be as general as possible, the possibility will be allowed that R_μ and L_μ couple to different condensates $\langle \overline{g} g \rangle$ and $\langle \overline{f} f \rangle$, respectively, though these condensates need not be distinct.

It was shown in Chapter 8 that the contribution of a left-handed gauge boson to the self-energy of a light quark necessarily has the structure $(1 + \gamma_5)\not{p}$. Similarly, the contribution of a right handed gauge boson has the structure $(1 - \gamma_5)\not{p}$. The question addressed in this Chapter is whether such self-energies, upon inclusion of the condensate of a heavy fermion, can induce dynamical masses for the light fermions, even though the light fermions are protected from acquiring masses via *purely* perturbative processes.

9.2 Renormalization at the Physical Mass-Shell

The model of chiral gauge interactions, as discussed above, leads to self-energy contributions of the form

$$\Sigma(\not{p}) = \frac{1}{2}\sigma_L(p^2)(1 + \gamma_5)\not{p} + \frac{1}{2}\sigma_R(p^2)(1 - \gamma_5)\not{p}\,. \tag{9.2.1}$$

The components of the self-energy are

$$
\begin{aligned}
\Sigma_{1,ij}(p^2) &= 0\,, \\
\Sigma_{\gamma,ij}(p^2) &= \frac{1}{2}\left[\sigma_{L,ij}(p^2) + \sigma_{R,ij}(p^2)\right], \\
\Sigma_{5,ij}(p^2) &= 0\,, \\
\Sigma_{5\gamma,ij}(p^2) &= \frac{1}{2}\left[\sigma_{L,ij}(p^2) - \sigma_{R,ij}(p^2)\right].
\end{aligned}
\tag{9.2.2}
$$

The inverse propagator is given in equation (3.1.8):

$$
\begin{aligned}
\mathbf{K}(\not p) = \frac{1}{2}&\left[|\mathbf{Z}_L^{1/2}|^2 + |\mathbf{Z}_R^{1/2}|^2 + \sigma_L(p^2) + \sigma_R(p^2)\right]\not p \\
&+ \frac{1}{2}\left[|\mathbf{Z}_L^{1/2}|^2 - |\mathbf{Z}_R^{1/2}|^2 + \sigma_L(p^2) - \sigma_R(p^2)\right]\gamma_5\not p\,.
\end{aligned}
\tag{9.2.3}
$$

The mass matrix $\mathbf{M}_0$ is zero because of the chiral symmetry. The renormalization conditions (3.2.14)–(3.2.16) result in the equations (3.2.21)-(3.2.23), which, in the present case, become

$$
\mu_i\left(|\mathbf{Z}_L^{1/2}|^2_{ii} + |\mathbf{Z}_R^{1/2}|^2_{ii}\right) + \mu_i\left[\sigma_{L,ii}(\mu_i^2) + \sigma_{R,ii}(\mu_i^2)\right] = 0\,, \tag{9.2.4}
$$

$$
|\mathbf{Z}_L^{1/2}|^2_{ii} - |\mathbf{Z}_R^{1/2}|^2_{ii} + \sigma_{L,ii}(\mu_i^2) - \sigma_{R,ii}(\mu_i^2) = 0\,, \tag{9.2.5}
$$

$$
|\mathbf{Z}_L^{1/2}|^2_{ii} + |\mathbf{Z}_R^{1/2}|^2_{ii} = 2 - \left[\sigma_{L,ii}(\mu_i^2) + \sigma_{R,ii}(\mu_i^2)\right] - 2\mu_i^2\left[\sigma'_{L,ii}(\mu_i^2) + \sigma'_{R,ii}(\mu_i^2)\right]. \tag{9.2.6}
$$

Of course, the $p^2 = 0$ mass-shell must remain a solution for the physical mass. If μ_i^2 is set equal to zero in equations (9.2.4)-(9.2.6), then (9.2.4) vanishes and (9.2.5) and (9.2.6) can be solved to obtain

$$
|\mathbf{Z}_L^{1/2}|^2_{ii} = 1 - \sigma_{L,ii}(0) \qquad |\mathbf{Z}_R^{1/2}|^2_{R,ii} = 1 - \sigma_{R,ii}(0)\,. \tag{9.2.7}
$$

Moreover, all of the off-diagonal equations (3.2.24)–(3.2.27) result in the trivial identity $0 = 0$. Indeed, equation (9.2.7) is just the usual purely perturbative solution in which self-energy effects are entirely absorbed in wave-function renormalization constants. Thus, the on-mass-shell renormalization procedure of Chapter 3 is fully consistent with retaining $\mu_i^2 = 0$ as the renormalization subtraction mass.

Now it is of interest to see if the renormalization conditions can be consistent with a *nonzero* subtraction mass. If $\mu_i^2 \neq 0$, then equation (9.2.6) can be multiplied by μ_i^2 and subtracted from (9.2.4) to obtain

$$1 = \mu_i^2 \left[\sigma'_{L,ii}(\mu_i^2) + \sigma'_{R,ii}(\mu_i^2)\right] . \tag{9.2.8}$$

Furthermore, μ_i can be cancelled from (9.2.4) and then (9.2.4) and (9.2.5) can be added and subtracted to obtain

$$\left|\mathbf{Z}_L^{1/2}\right|^2_{ii} = -\sigma_{L,ii}(\mu_i^2), \tag{9.2.9}$$

$$\left|\mathbf{Z}_R^{1/2}\right|^2_{ii} = -\sigma_{R,ii}(\mu_i^2). \tag{9.2.10}$$

This solution is nonperturbative: $\mathbf{Z}_L$ and $\mathbf{Z}_R$ are no longer unity to leading order. These renormalization conditions are possible as long as:

(i) both $\sigma_{L,ii}(\mu_i^2)$ and $\sigma_{R,ii}(\mu_i^2)$ are negative, and

(ii) the sum of the derivatives $\left[\sigma'_{L,ii}(\mu_i^2) + \sigma'_{R,ii}(\mu_i^2)\right]$ is positive. (9.2.11)

In order to determine whether these conditions are satisfied, the contribution of the heavy fermion condensate to the light quark self energies must be computed.

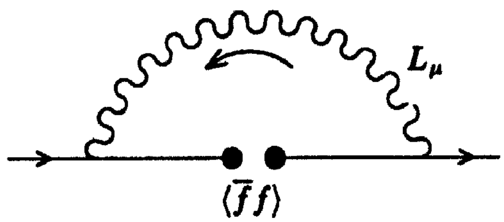


Figure 9.1 The $\langle \bar{f} f \rangle$ contribution to the light quark self-energy by exchange of a L_μ gauge boson.

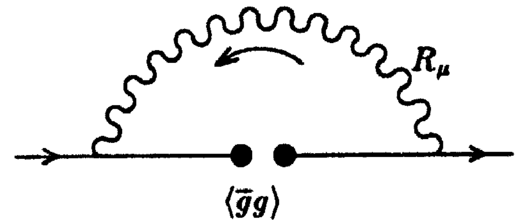


Figure 9.2 The $\langle \bar{g} g \rangle$ contribution to the light quark self-energy be exchange of a R_μ gauge boson.

9.3 Dynamical Generatio ı of Fermion Masses

It is shown below that the inverse propagator also vanishes at a nonzero value of μ^2. However, it will become clear that this result does not correspond to a physical mass.

Suppose that σ_L arises from the left-handed off-diagonal coupling of a massive gauge-boson L^μ to a fermion-antifermion condensate $\langle \bar{f} f \rangle$ (Figure 9.1). A calculation completely analogous to that leading to (7.2.19) yields a result of the form

$$\sigma_L(p^2) = -\frac{g_L^2 \langle \bar{f} f \rangle \mu_f}{M_L^2(\mu_F^2 - M_L^2)^2}\left[(\mu_f^2 - 2M_L^2) - \frac{(9M_L^4 - 4\mu_f^2 M_L^2 + \mu_f^4)}{3(\mu_f^2 - M_L^2)^2} p^2 + O(p^4)\right]. \tag{9.3.1}$$

From now on the flavour indices will be suppressed. The positive constant g_L^2 absorbs the square of the gauge coupling constant, numerical constants associated with the L-i-f vertex, as well as the product of the mixing angle factors $(V^\dagger)_{if} V_{fi}$. We see from (9.3.1) that for $\mu^2 << \mu_f^2$, M_L^2

$$\sigma_L(\mu^2) = -\frac{g_L^2 \langle \bar{f} f \rangle \mu_f (\mu_f^2 - 2M_L^2)}{M_L^2(\mu_f^2 - M_L^2)^2}, \tag{9.3.2}$$

$$\sigma_L(\mu^2) = \frac{g_L^2 \langle \bar{f} f \rangle \mu_f (9M_L^4 - 4\mu_f^2 M_L^2 + \mu_f^4)}{3M_L^2(\mu_f^2 - M_L^2)^4}. \tag{9.3.3}$$

Similar expressions can be obtained for σ_R and σ'_R based on the right-handed off-diagonal coupling of a gauge boson R^μ to $\langle \overline{g}g \rangle$ (Figure 9.2), with $\mu^2 << \mu_g^2, M_R^2$.

In order to satisfy condition (i) of (9.2.11), $\sigma_L(\mu^2)$ must be negative. From (9.3.2) this will be the case if either $\langle \overline{f}f \rangle$ is negative and $(\mu_f^2 - 2M_L^2)$ are positive, or $\langle \overline{f}f \rangle$ is positive and $(\mu_f^2 - 2M_L^2)$ are negative. Therefore, if $\langle \overline{f}f \rangle$ is negative then $\mu_f > \sqrt{2}M_L$, and if $\langle \overline{f}f \rangle$ is positive then $\mu_f < \sqrt{2}M_L$.

In order to satisfy condition (ii) of (9.2.11), either $\sigma'_L(\mu^2)$ or $\sigma'_R(\mu^2)$ must be positive. If one of them is positive and one of them negative, then the one that is positive must be greater than the one that is negative. Suppose, $\sigma'_L(\mu^2)$ is the one that is negative. Because the term $(9M_L^4 - 4\mu_f^2 M_L^2 + \mu_f^4) = (\mu_f^2 - 2M_L^2)^2 + 5M_L^4$ is positive definite, $\sigma'_L(\mu^2)$ can be negative only if $\langle \overline{f}f \rangle$ is negative. The condition (i) imposed the constraint that if $\langle \overline{f}f \rangle$ is negative then $\mu_f > \sqrt{2}M_L$. This means that the heavy fermion is heavier than the gauge boson, which is suggestive of models where dynamical symmetry breaking is driven by very massive fermions.

Substitution of (9.3.3) and the corresponding equation for $\sigma_R(\mu^2)$ into (9.2.8) yields

$$\mu^2 = \frac{1}{\dfrac{g_L^2 \langle \overline{f}f \rangle \mu_f (9M_L^4 - 4\mu_f^2 M_L^2 + \mu_f^4)}{3M_L^2(\mu_f^2 - M_L^2)^4} + \dfrac{g_R^2 \langle \overline{g}g \rangle \mu_f (9M_R^4 - 4\mu_f^2 M_R^2 + \mu_f^4)}{3M_R^2(\mu_f^2 - M_R^2)^4}} . \tag{9.3.4}$$

The dynamical mass μ is inversely proportional to the fermion condensates, thus linking small induced fermion current masses to the presence of very large condensates of very massive fermions.

However, there are two reasons why the mass determined by (9.3.4) should not be regarded as the physical mass of the light fermion. First, purely left-and right-handed interactions cannot connect a left-handed fermion going in to a right-handed fermion coming out, which is required in order to generate a mass term. The pole could, however, be shifted from zero by adding a term to the interaction that makes it not purely left-handed or right-handed. Second, the mass in (9.3.4) is inversley proportional to the couplings g_L and g_R. Because the interaction is generating the mass, one would expect the mass to vanish as the interation is turned off ($g_L, g_R \to 0$). However, in (9.3.4), the mass becomes infinite as the interaction is turned off. The mass is also inversely proportional to the condensate and becomes infinite as the condensate vanishes. Therefore, the solution (9.3.4) should not be regarded as the physical mass of the light fermion; the only meaningful solution is $\mu^2 = 0$.

SUMMARY

The renormalization of electroweak theory can be performed consistently at a mass that differs from the Lagrangian mass. The renormalization point is interpreted as the constituent quark mass μ and the Lagrangian mass is interpreted as the current mass m. The renormalization yields an equation that determines the mass difference $(\mu - m)$ in terms of the nonperturbative component of the self-energy.

The use of the constituent mass necessitates the introduction of nonperturbative contributions to the quark self-energies. These contributions arise from the ground state expectation value of the normal-ordered product of fermionic fields. The dimension-3 quark condensate $\langle\overline{q}q\rangle$ dominates the expansion of the nonperturbative propagator $\langle\tilde{0}| : \psi(x)\overline{\psi}(0) : |\tilde{0}\rangle$. Therefore, the $\langle\overline{q}q\rangle$ contribution to the self-energy dominates the mass difference $(\mu - m)$. The $\langle\overline{q}q\rangle$ contribution of the strong interactions dominates the $\langle\overline{q}q\rangle$ contribution of the electroweak interactions. This calculation assumes that the t-quark condensate $\langle\bar{t}t\rangle$ is negligible due to the heavy quark expansion.

However, some recently proposed models that involve interactions beyond the standard model allow for a very large t-quark condensate. If $\langle\bar{t}t\rangle$ is very large then it will have an effect on processes that are strictly within the context of the standard model.

The magnitude of a large t-quark condensate is limited by the mass difference between the u- and d-quarks. The $\langle\bar{t}t\rangle$ contribution to the self-energy is at most of

the same order as the magnitude of the u–d mass difference. From this restriction, $|\langle\bar{t}t\rangle|$ can be at most 105.62×10^9 GeV^3, which occurs at the lowest possible value for the CKM parameter $V_{td} = 0.003$ and a t–quark mass of $\mu_t = 120$ GeV.

The magnitude of a large t–quark condensate is also restricted by its contribution to the strangeness–changing amplitude of $K \rightarrow 2\pi$ decays. The off–diagonal transition of an s–quark to a d–quark must contribute to the strangeness–changing amplitude $a_{1/2}$. The $\langle\bar{t}t\rangle$ condensate contributes to the amplitude $a_{1/2}$ because it appears in the s–d transition with a W^+ exchange. From this restriction, $|\langle\bar{t}t\rangle|$ can be at most 36.55×10^8 GeV^3, which occurs at the lowest possible values for the CKM parameters $V_{td} = 0.003$ and $V_{ts} = 0.029$ and a t–quark mass of $\mu_t = 160$ GeV.

The u–d mass difference and the strangeness–changing amplitude of $K \rightarrow 2\pi$ decays restrict the magnitude of $\langle\bar{t}t\rangle$ to be less than 10^9–10^{11} GeV^3. Therefore, a t–quark condensate referenced to the Planck mass scale is not possible within the context of the particular scheme assumed in this thesis.

Finally, it is possible within the context of a larger model for which light quarks are protected from acquiring mass from purely perturbative effects for a heavy condensate to dynamically generate the current masses of light fermions. Such a theory must have intergenerational mixing in order for the heavy fermion to have some way of coupling to the light fermions. The theory must also have both a left–handed and a right–handed gauge interactions in order for the renormalization to be consistent. Under these circumstances the large condensate of the heavy fermion will be inversely proportional to the light quark mass. However, this solution is not meaningful because it is impossible to connect a left-handed field to a right-handed field, which is required to generate a mass term, using purely left-handed and right-handed interations. Also, the nonzero solution is inversely proportional to the coupling and the condensate, implying that the mass becomes infinite as the interaction is turned off. Therefore, although a nonzero solution ($\mu^2 \neq 0$) exists, the only meaningful solution is the zero solution ($\mu^2 = 0$).

APPENDIX A
CONVENTIONS AND IDENTITIES

The metric tensor:

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{A.1}$$

The chiral representation for the gamma matrices:

$$\gamma^0 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix} \quad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \tag{A.2}$$

where $\vec{\sigma}$ are the Pauli matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{A.3}$$

and I is a 2×2 identity matrix. The left and right projection operators:

$$L = \frac{(1-\gamma_5)}{2} \qquad R = \frac{(1+\gamma_5)}{2} \tag{A.4}$$

The Fermi and the weak coupling are related by $G_F\sqrt{2} = g_W^2/(8M_W^2)$. The values for the parameters of the electroweak theory are:

weak goupling	$g_w = 0.65$
Fermi coupling	$G_F = 1.166 \times 10^{-5}$ GeV^{-2}
pion decay constant	$f_\pi = 0.132$ GeV
pion mass	$m_\pi = 0.135$ GeV
Kaon mass	$m_K = 0.495$ GeV
W mass	$M_W = 80$ GeV
Z mass	$M_Z = 92$ GeV

The CKM matrix:[42]

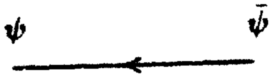
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$
$$= \begin{pmatrix} 0.975 \text{ to } 0.976 & 0.218 \text{ to } 0.224 & 0.001 \text{ to } 0.007 \\ 0.218 \text{ to } 0.224 & 0.973 \text{ to } 0.975 & 0.030 \text{ to } 0.058 \\ 0.003 \text{ to } 0.019 & 0.029 \text{ to } 0.058 & 0.998 \text{ to } 0.999 \end{pmatrix}. \quad \text{(A.5)}$$

Contraction identities for the gamma matrices:[32]

$$\begin{aligned} \gamma^\lambda \gamma_\lambda &= 4 \\ \gamma^\lambda \gamma^\mu \gamma_\lambda &= -2\gamma^\mu \\ \gamma^\lambda \gamma^\mu \gamma^\nu \gamma_\lambda &= 4g^{\mu\nu} \\ \gamma^\lambda \gamma^\mu \gamma^\nu \gamma^\rho \gamma_\lambda &= -2\gamma^\rho \gamma^\nu \gamma^\mu \\ \gamma^\lambda \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_\lambda &= 2(\gamma^\sigma \gamma^\mu \gamma^\nu \gamma^\rho + \gamma^\rho \gamma^\nu \gamma^\mu \gamma^\sigma) \end{aligned} \quad \text{(A.6)}$$

APPENDIX B
FEYNMAN RULES

The propagators:[3]

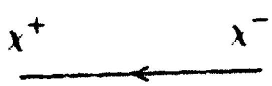


$$\frac{1}{m - \not{p} - i\epsilon} \tag{B.1}$$

$$\frac{1}{p^2 - M_W^2 + i\epsilon}\left[g^{\mu\nu} - (1-\alpha_W)\frac{p^\mu p^\nu}{p^2 - \alpha_W M_W^2 + i\epsilon}\right] \tag{B.2}$$

$$\frac{1}{p^2 - M_Z^2 + i\epsilon}\left[g^{\mu\nu} - (1-\alpha_Z)\frac{p^\mu p^\nu}{p^2 - \alpha_Z M_Z^2 + i\epsilon}\right] \tag{B.3}$$

$$\frac{1}{p^2 + i\epsilon}\left[g^{\mu\nu} - (1-\alpha_A)\frac{p^\mu p^\nu}{p^2 + i\epsilon}\right] \tag{B.4}$$

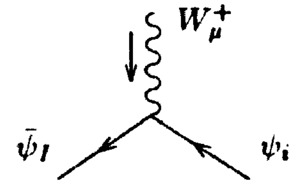


$$\frac{1}{\alpha_W M_W^2 - p^2 - i\epsilon} \tag{B.5}$$

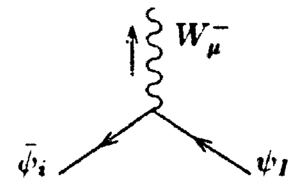
$$\frac{1}{\alpha_Z M_Z^2 - p^2 - i\epsilon} \tag{B.6}$$

$$\frac{1}{m_\phi - p^2 - i\epsilon} \tag{B.7}$$

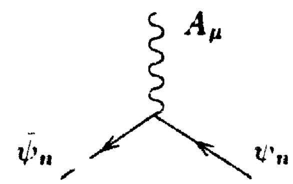
The fermion–fermion–gauge boson vertices:[3]



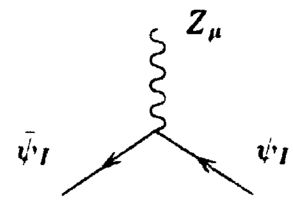
$$\frac{g_w}{2\sqrt{2}} V_{Ii} \gamma_\mu (1 - \gamma_5) \tag{B.8}$$



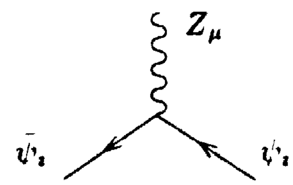
$$\frac{g_w}{2\sqrt{2}} (V^\dagger)_{iI} \gamma_\mu (1 - \gamma_5) \tag{B.9}$$



$$e Q_n \gamma_\mu \qquad n = i, I \tag{B.10}$$



$$-\frac{g_w M_Z^2}{4 M_W} \gamma_\mu (\lambda_I + \gamma_5) \tag{B.11}$$

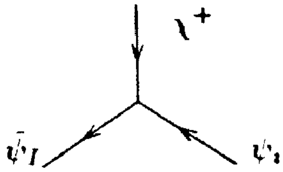


$$\frac{g_w M_Z^2}{4 M_W} \gamma_\mu (\lambda_i + \gamma_5) \tag{B.12}$$

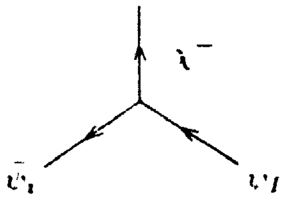
where

$$\lambda_i = -1 + \frac{4(M_Z^2 - M_W^2)}{3 M_Z^2} \qquad \text{and} \qquad \lambda_I = -1 + \frac{8(M_Z^2 - M_W^2)}{3 M_Z^2}. \tag{B.13}$$

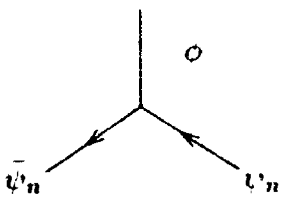
The fermion–fermion–scalar vertices:[3]



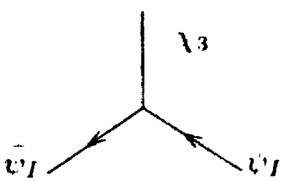
$$-\frac{ig_w}{2\sqrt{2}M_W}V_{Ii}\left[(m_i - m_i) + (m_i + m_I)\gamma_5\right] \qquad \text{(B.14)}$$



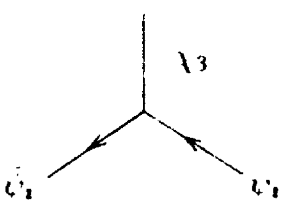
$$-\frac{ig_w}{2\sqrt{2}M_W}(V^\dagger)_{iI}\left[(m_I - m_i) + (m_I + m_i)\right] \qquad \text{(B.15)}$$



$$-\frac{g_w m_n}{2M_W} \qquad n = i, I \qquad \text{(B.16)}$$



$$-\frac{ig_w m_I}{2M_W}\gamma_5 \qquad \text{(B.17)}$$



$$\frac{ig_w m_i}{2M_W}\gamma_5 \qquad \text{(B.18)}$$

APPENDIX C
THE SERIES METHOD

To compute the self-energies of Chapter 6, the following identity is employed:

$$\int \frac{d^4 q}{(2\pi)^4 i} S_n^{NP}(p-q) f(q) = i\langle\overline{\psi}_n \psi_n\rangle \sum_{j=0}^{\infty} C_j \mu_n^j \left[-\gamma \cdot \frac{\partial}{\partial p}\right]^j f(p), \tag{C.1}$$

where $S_n^{NP}(p-q)$ is the nonperturbative propagator[11] and $f(p)$ is an arbitrary function of p. The quark condensate part of the nonperturbative propagator is represented by a pair of dots on the internal quark line in Figures 6.1 and 6.2. The subscript n represents flavour and it is either uppercase (i.e., $I, J, \ldots$) for charge $+2/3$ quarks, or lowercase (i.e., $i, j, \ldots$) for charge $-1/3$ quarks. The coefficients C_j are defined by[9,11]

$$C_j = \begin{cases} \left[3(j/2)!(j/2+1)!4^{(j/2+1)}\right]^{-1} & \text{j even} \\ \left[6(j/2-1/2)!(j/2+3/2)!4^{(j+1)/2}\right]^{-1} & \text{j odd} \end{cases} \quad . \tag{C.2}$$

For various forms of $f(p)$ that do not contain nonzero poles, one obtains[37]

$$\sum_{j=0}^{\infty} C_j \mu_n^j \left[-\gamma \cdot \frac{\partial}{\partial p}\right]^j \frac{1}{p^2} = \frac{1}{12p^2}\left[1 + \frac{\mu_n \not{p}}{2p^2}\right], \tag{C.3}$$

$$\gamma^\mu \sum_{j=0}^{\infty} C_j \mu_n^j \left[-\gamma \cdot \frac{\partial}{\partial p}\right]^j \frac{g_{\mu\nu}}{p^2}\gamma^\nu = \frac{1}{12p^2}\left[4 - \frac{\mu_n \not{p}}{p^2}\right], \tag{C.4}$$

$$\gamma^\mu \sum_{j=0}^{\infty} C_j \mu_n^j \left[-\gamma \cdot \frac{\partial}{\partial p}\right]^j \frac{p_\mu p_\nu}{p^2}\gamma^\nu = \frac{1}{12p^2}\left[p^2 + \frac{(\mu_n^2 - 4p^2)\mu_n \not{p}}{2p^2}\right], \tag{C.5}$$

$$\gamma^\mu \sum_{j=0}^{\infty} C_j \mu_n^j \left[-\gamma \cdot \frac{\partial}{\partial p}\right]^j \frac{p_\mu p_\nu}{p^4}\gamma^\nu = \frac{1}{12p^2}\left[1 - \frac{\mu_n \not{p}}{p^2}\right]. \tag{C.6}$$

For the cases in which $f(p)$ contains a mass pole, the requisite identities are[37]

$$\sum_{j=0}^{\infty} C_j \mu_n^j \left[-\gamma \cdot \frac{\partial}{\partial p}\right]^j \frac{1}{p^2 - M^2}$$
$$= \frac{1}{12} \sum_{l=0}^{\infty} \sum_{k=0}^{l+1} A(l,k) \frac{(\mu_n^2)^k (p^2)^{l-k}}{(M^2)^{l+1}} \frac{(l+1-k)}{(l+2)} \left[-1 + \frac{(l-k)\mu_n \not{p}}{(k+2)p^2}\right],$$
$$\approx -\frac{1}{12M^2} + O\left(\frac{1}{M^4}\right), \qquad \text{(C.7)}$$

$$\gamma^\mu \sum_{j=0}^{\infty} C_j \mu_n^j \left[-\gamma \cdot \frac{\partial}{\partial p}\right]^j \frac{g_{\mu\nu}}{p^2 - M^2} \gamma^\nu$$
$$= -\frac{1}{6} \sum_{l=0}^{\infty} \sum_{k=0}^{l+1} A(l,k) \frac{(\mu_n^2)^k (p^2)^{l-k}}{(M^2)^{l+1}} \frac{(l+1-k)}{(l+2)} \left[2 + \frac{(l-k)\mu_n \not{p}}{(k+2)p^2}\right],$$
$$\approx -\frac{1}{3M^2} + O\left(\frac{1}{M^4}\right). \qquad \text{(C.8)}$$

$$\gamma^\mu \sum_{j=0}^{\infty} C_j \mu_n^j \left[-\gamma \cdot \frac{\partial}{\partial p}\right]^j \frac{p_\mu p_\nu / M^2}{p^2 - M^2} \gamma^\nu$$
$$= -\frac{p^2}{12M^2} \sum_{l=0}^{\infty} \sum_{k=0}^{l+1} A(l,k) \frac{(\mu_n^2)^k (p^2)^{l-k}}{(M^2)^{l+1}} \frac{(l+1)}{(l+2-k)}$$
$$\times \left[1 - \frac{(l+4)(l+1-k)\mu_n \not{p}}{(l+1)(k+2)p^2}\right],$$
$$\approx 0 + O\left(\frac{1}{M^4}\right). \qquad \text{(C.9)}$$

where the coefficient $A(l,k)$ is defined to be

$$A(l,k) = \frac{l!(l+2)!}{\left[(l+1-k)!\right]^2 k!(k+1)!}. \qquad \text{(C.10)}$$

The right hand sides of (C.7)–(C.9) include the order $1/M^2$ approximation appropriate for computing the self energies when M is either M_Z, M_W or m_ϕ.

APPENDIX D
The Fourier Method

The Fourier transform $\mathcal{F}_t(k)$ of the ground state expectation value of the normal-ordered product of fermion fields is defined in (7.2.11)[17]

$$\int d^4k\, e^{-ik\cdot x}\mathcal{F}_t(k) = -\frac{\langle\bar{t}t\rangle J_1(\mu_t\sqrt{x^2})}{6\mu_t^2\sqrt{x^2}}\,. \tag{D.1}$$

The first useful identity is derived by taking the $x \to 0$ limit of (D.1)

$$\int d^4k\, \mathcal{F}_t(k) = -\frac{\langle\bar{t}t\rangle}{12\mu_t}\,. \tag{D.2}$$

The following integral appears in (7.2.16)

$$I(p^2) = \int d^4k\, \frac{\mathcal{F}_t(k)}{(p-k)^2 - M_W^2 + i\epsilon}\,. \tag{D.3}$$

Use the identity

$$\frac{1}{x+i\epsilon} = -i\int_0^\infty d\eta\, e^{i\eta(x+i\epsilon)} \tag{D.4}$$

to exponentiate the denominator of (D.3)[17]

$$I(p^2) = -i\int_0^\infty d\eta\, e^{-\eta\left[\epsilon - i(p^2+\mu_t^2-M_W^2)\right]}\int d^4k\, e^{-i(2\eta p)\cdot k}\mathcal{F}_t(k)\,. \tag{D.5}$$

The relation (7.2.13) was also used to derive (D.5). Use the definition (D.1) with $x = 2\eta p$ to obtain

$$I(p^2) = \frac{i\langle\bar{t}t\rangle}{12\mu_t^2\sqrt{p^2}}\int_0^\infty \frac{d\eta}{\eta}e^{-\eta\left[\epsilon - i(p^2+\mu_t^2-M_W^2)\right]}J_1(2\mu\sqrt{p^2}\eta)\,. \tag{D.6}$$

Equation 6.623.3 on page 712 of Gradshteyn and Ryzhik[50] is

$$\int_0^\infty \frac{\mathrm{d}\eta}{\eta}\epsilon^{-\eta\alpha} J_\nu(\beta\eta) = \frac{(\sqrt{\alpha^2+\beta^2}-\alpha)^\nu}{\nu\beta^\nu}\,, \tag{D.7}$$

with the conditions

$$\mathrm{Re}\,\nu > 0 \qquad \text{and} \qquad \mathrm{Re}\,\alpha > |\mathrm{Im}\,\beta|\,. \tag{D.8}$$

If $\nu = 1$ then the first condition is satisfied. If $\alpha = \epsilon - i(p^2 + \mu_t^2 - M_W^2)$ and $\beta = 2\mu_t\sqrt{p^2}$ then the second condition reduces to $\epsilon > 0$, which is true by definition of ϵ. With these definitions of ν, α and β, equation (D.7) can be used in (D.6) to obtain

$$I(p^2) = \frac{i\langle\bar{t}t\rangle}{12\mu_t^2\sqrt{p^2}}\frac{\sqrt{(-i)^2(p^2+\mu_t^2-M_W^2)^2+4\mu_t^2p^2}+i(p^2+\mu_t^2-M_W^2)}{2\mu_t\sqrt{p^2}}\,. \tag{D.9}$$

One must be careful when extracting the factor of $(-i)^2$ from the radical to choose the correct branch

$$I(p^2) = \frac{i\langle\bar{t}t\rangle}{12\mu_t^2\sqrt{p^2}}\frac{-i\sqrt{(p^2+\mu_t^2-M_W^2)^2-4\mu_t^2p^2}+i(p^2+\mu_t^2-M_W^2)}{2\mu_t\sqrt{p^2}}\,. \tag{D.10}$$

This gives the solution

$$I(p^2) = -\frac{\langle\bar{t}t\rangle}{24\mu_t^3p^2}\left[(p^2+\mu_t^2-M_W^2)-\sqrt{(p^2+\mu_t^2-M_W^2)^2-4\mu_t^2p^2}\right]. \tag{D.11}$$

This function is valid only for those values of p^2 such that the quantity underneath the radical sign is positive. In order to find the allowed domain of the function, it is convenient to rewrite the quantity under the radical sign

$$\begin{aligned} I(p^2) &= \int \mathrm{d}^4k\,\frac{\mathcal{F}_t(k)}{(p-k)^2-M_W^2} \\ &= -\frac{\langle\bar{t}t\rangle}{24\mu_t^3p^2}\left[(p^2+\mu_t^2-M_W^2)-\sqrt{(p^2-\mu_t^2-M_W^2)^2-4\mu_t^2M_W^2}\right]. \end{aligned} \tag{D.12}$$

The momentum p^2 is restricted by the condition

$$(p^2 - \mu_t^2 - M_W^2)^2 \geq 4\mu_t^2 M_W^2 \tag{D.13}$$

to two allowable domains

$$p^2 \geq (\mu_t + M_W)^2 \qquad \text{and} \qquad p^2 \leq (\mu_t - M_W)^2 . \tag{D.14}$$

Another useful integral is

$$J(p^2) = \int \mathrm{d}^4 k \, \frac{\mathcal{F}_t(k)\, p \cdot k}{(p-k)^2 - M_W^2} . \tag{D.15}$$

Completing the square in the numerator by adding and subtracting terms, one obtains

$$J(p^2) = -\frac{1}{2} \int \mathrm{d}^4 k \, \mathcal{F}_t(k) \frac{(p-k)^2 - M_W^2 - (p^2 + \mu_t^2 - M_W^2)}{(p-k)^2 - M_W^2} . \tag{D.16}$$

This can be split into two integrals

$$J(p^2) = -\frac{1}{2} \int \mathrm{d}^4 k \, \mathcal{F}_t(k) + \frac{1}{2}(p^2 + \mu_t^2 - M_W^2) \int \mathrm{d}^4 k \, \frac{\mathcal{F}_t(k)}{(p-k)^2 - M_W^2} . \tag{D.17}$$

The solution to the first integral is given in (D.2), and the solution to the second integral is given in (D.12). Thus, (D.17) is

$$J(p^2) = \frac{\langle \bar{t} t \rangle}{24 \mu_t} \Bigg\{ 1 - \frac{(p^2 + \mu - M_W^2)}{2\mu_t^2 p^2} \times \left[(p^2 + \mu_t^2 - M_W^2) - \sqrt{(p^2 - \mu_t^2 - M_W^2)^2 - 4\mu_t^2 M_W^2} \right] \Bigg\} . \tag{D.18}$$

Using the definition (D.12) for $I(p^2)$ this can be rewritten

$$J(p^2) = \int d^4k \, \frac{\mathcal{F}_t(k)\, p\cdot k}{(p-k)^2 - M_W^2} = \frac{\langle \bar{t}t\rangle}{24\mu_t} + \frac{1}{2}(p^2 + \mu_t^2 - M_W^2) I(p^2)\,. \tag{D.19}$$

The function $J(p^2)$ has the domain given in (D.14)

One more identity that is used in Section 7.2 is (7.2.15). To prove this identity, note that the only four vector available to carry a Dirac index is the external momentum p. Therefore,

$$\int d^4k \, \frac{\mathcal{F}_t(k) \not{k}}{(p-k)^2 - M_W^2} = \not{p}\, G(p^2)\,, \tag{D.20}$$

Where $G(p^2)$ must now be determined. Multiplying both sides of equation (D.20) on the left and then on the right by $\not{p}$ and adding the two, one obtains

$$\int d^4k \, \frac{\mathcal{F}_t(k)(\not{p}\not{k} + \not{k}\not{p})}{(p-k)^2 - M_W^2} = 2p^2 G(p^2)\,. \tag{D.21}$$

Subsitution of (D.21) back into (D.20) yields the required identity

$$\int d^4k \, \frac{\mathcal{F}_t(k) \not{k}}{(p-k)^2 - M_W^2} = \frac{\not{p}}{p^2} \int d^4k \, \frac{\mathcal{F}_t(k)\, p\cdot k}{(p-k)^2 - M_W^2}\,. \tag{D.22}$$

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